FEASIBILITY STUDY OF NATURAL CIRCULATION BOILING WATER REACTOR BY EXAMINING CORE STABILITY REQUIREMENT

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by

MADHUSUDEN AGRAWAL

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CERTIFICATE

This is to certify that the present work entitled, " FEASIBILITY STUDY OF NATURAL CIRCULATION BOILING WATER REACTOR BY EXAMINING CORE STABILITY REQUIREMENT " has been carried out by Mr. Madhusuden Agrawal under my supervision and has not been submitted elsewhere for a degree.

K. Sri Ram

March, 1994

Dr. K.Sriram
Professor
Department of Mechanical Engineering
Indian Institute of Technology
Kanpur - 208016
India

ABSTRACT

Conventional nuclear reactors suffer from many drawbacks such as complex reactor design resulting in frequent safety problems, overall high cost of power generation sue to long construction times and retrofittings to comply with safety stipulations, etc. To overcome these problems, recently there has been considerable interest to develop an improved nuclear reactor. The Boiling Water Reactor using natural circulation as a mode for coolant flow is one of such new reactor which has the potential for overcoming many safety related problems due to elimination of external pumps and simplify the reactor vessel design considerably.

The present thesis analyses the feasibility of the natural circulation BWR for higher power rates satisfying one of the thermal hydraulic requirements, that is the core stability. The study shows that the natural circulation BWR for powers >1000 MW is indeed feasible.

ACKNOWLEDGEMENT

With deepest sense of gratitude, I acknowledge the key role played by my thesis supervisor , Dr. K.Sriram , in bringing about the success of this thesis. It is only due to his healthy guidance and intellectual support, that this work has been able to reach in its final form.

I express my thanks to Ranjan, Kanu, Chetan, and other friends for giving all kinds of help and encouragement. I owe a lot to them.

March,1994. Madhusuden Agrawal

CONTENTS

LIST OF FIGURES

LIST OF TABLES

SYMBOLS

M	:	Mass of the fuel rod (Kg).
p	:	Operating pressure of reactor (MPa).
C_f	:	Specific heat of fuel material (KJ/Kg/ °C).
T_f	:	Fuel temperature (°C).
T_c	:	Coolant temperature (°C).
T_{f0}	:	Steady state fuel temperature (°C).
T_{c0}	:	Steady state coolant temperature (°C).
Π_f	:	Perturbation in fuel temperature (°C).
Π_c	:	Perturbation in coolant temperature (°C)
q	:	Fuel power density (KW/l).
q"	:	Heat flux (MW/m^2).
V	:	Volume of fuel rod (m^3).
A	:	Surface area of fuel rod (m^2).
q_0	:	Steady state fuel power density (KW/l).
q''_0	:	Steady state heat flux (MW/m^2).
Q	:	Non-dimensional fuel power density.
Q"	:	Non-dimensional heat flux.
h	:	Heat transfer coefficient (W/m^2/ °C).
h_0	:	Steady state heat transfer coefficient (W/m^2/ °C).
H	:	Non-dimensional heat transfer coefficient.
k	:	Thermal conductivity of coolant (W/m/ °C).
L	:	Length of fuel rod (m).
Re	:	Reynolds number.
Pr	:	Prank number.

G	:	Coolant flow rate (Kg/s).
G_0	:	Steady state coolant flow rate (Kg/s).
$\mathbb{G}$	:	Non-dimensional perturbation in coolant flow.
A_c	:	Cross-section area for coolant flow (m^2).
ρ_c	:	coolant density (Kg/m^3).
C_c	:	Specific heat of coolant (J/Kg/ °C).
A_p	:	Perimeter of the fuel rod (m).
λ	:	Non-boiling height (m).
λ_0	:	Steady state non-boiling height (m).
λ'	:	Perturbation in non-boiling height (m).
v	:	Coolant flow velocity in two phase region (m/s).
v_0	:	Steady state two phase coolant flow velocity (m/s).
v'	:	Perturbation in two phase coolant flow velocity(m/s)
v_1	:	Coolant flow velocity in single phase region (m/s).
v_{10}	:	Steady state single phase coolant flow velocity(m/s)
v_1'	:	Perturbation in single phase coolant flow velocity.
Ω	:	Characteristic frequency of phase change (/s).
v_{fg}	:	Change in specific volume of coolant at phase change (m^3/Kg).
v_f	:	Specific volume of coolant in liquid phase (m^3/Kg).
h_{fg}	:	Latent heat of coolant (KJ/Kg).
h_f	:	Enthalapy of coolant in liquid phase (KJ/Kg).
x	:	quality of coolant.
x_0	:	Steady state quality of coolant.
x'	:	Perturbation in coolant quality.
α	:	Void fraction.
α'	:	Perturbation in void fraction.
α'_{avg}	:	Perturbation in core average void fraction.

ρ_f : Coolant density in liquid phase (Kg/m^3).

ρ_g : Coolant density in vapor phase (Kg/m^3).

ρ_H : Density of two phase mixture (Kg/m^3).

ρ_{HO} : Steady state density of two phase mixture (Kg/m^3).

ρ'_{HO} : Perturbation in two phase mixture density (Kg/m^3).

ν : Time required for a fluid particle to loss its subcooling (s).

h_i : Enthalpy of coolant at core entrance (KJ/Kg).

z : Axial coordinate referenced to the core inlet (m).

y : Axial coordinate referenced to the boiling boundary (m).

t : Time variable (s).

t_0 : Time at which fluid particle crosses the boiling boundary (s).

t_{ex} : Time at fluid particle reaches at the top of core (s).

τ : Time variable referenced to t_0 (s).

τ' : Perturbation in τ (s).

τ_{ex} : Time required by fluid particle to pass from the two phase region (s).

f : Friction factor.

D_h : Hydraulic mean diameter (m).

g : Gravitational acceleration (m/s^2).

p_1 : Pressure drop in single phase region (MPa).

P_1 : Perturbation in pressure drop in single phase region (MPa).

p_2 : Pressure drop in two phase region (MPa).

P_2 : Perturbation in pressure drop in two phase

region (MPa).

β : Delayed neutron fraction.

λ_1 : Decay constant of precursor (/s).

Λ : Average life of prompt neutron (s).

\$: Non-dimensional reactivity of fuel material.

K_v : Void reactivity coefficient (\$/ % void).

DR : Decay ratio.

Δp_{loss} : Total pressure loss in core (MPa).

Δp_f : Total frictional pressure loss in core (MPa).

Δp_a : Total acceleration pressure loss in core (MPa).

$\Delta p_{c,e}$: Total pressure loss due to resistance to flow at abrupt are changes (MPa).

Δp_d : Total driving pressure (MPa).

ρ_{dc} : Coolant density in down comer (Kg/m^3).

ρ_{ex} : Coolant density at channel exit (Kg/m^3).

x_{ex} : Coolant quality at exit.

$\bar{\rho}$: Average coolant density in channel (Kg/m^3).

L_R : Riser height (m).

NOTE : Terms having bar($\bar{}$) at top of any variables represents the Laplace Transform of that variable.

CHAPTER 1

INTRODUCTION

The 1990s' has witnessed the spawning of new nuclear reactor designs in US and Japan which span the technological range from evolutionary light water and heavy water, through light water passive and gas cooled, to liquid metal fast breeder.

The likelihood of successful development and operation of any individual reactor design depends on a mix of both ' hard ' and 'soft ' variables. The hard variables are the more objective factors such as actual design characteristics, cost, length of technology leap, and degree of research and development support by the government and industrial organisations.

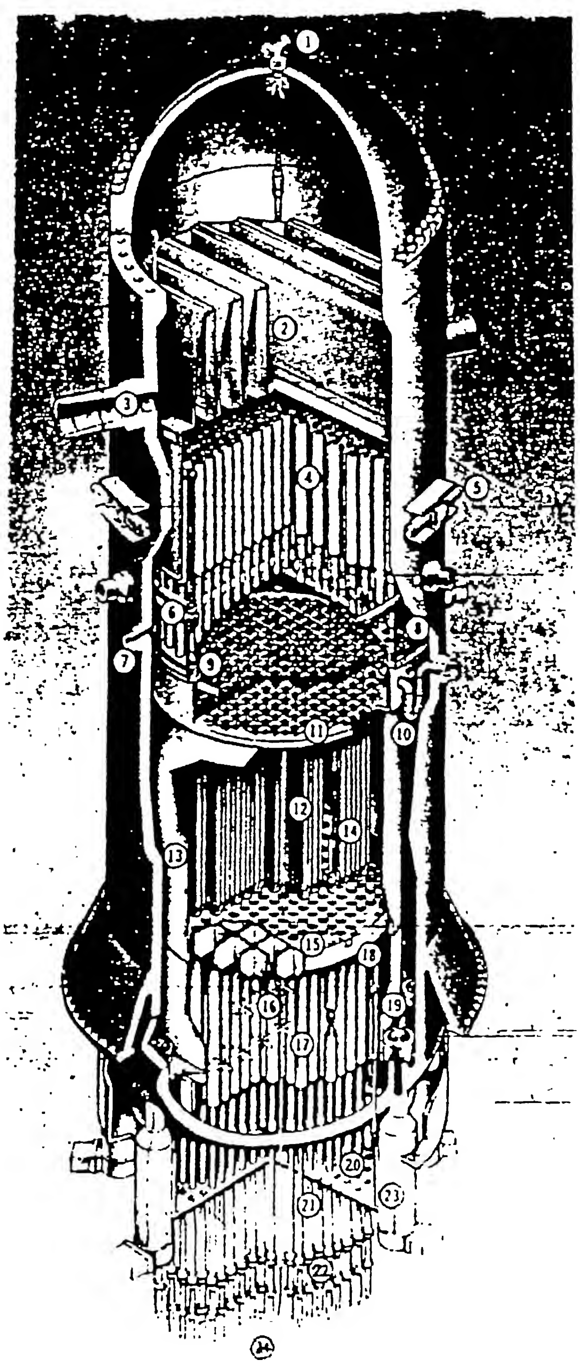
The soft variables are more ambiguous and depend more on so called ' public perception ' rather than on facts. They may involve imponderables like attitude towards carbon dioxide production and global warming, and alternative power technologies. Other factors are more nation specific, including degree of political opposition to siting of nuclear plants or radiation waste storage sites, public ambivalence towards the unknowns of the new technology versus the familiar one and the regulatory procedures. Yet other soft variables are utility specific such as size of available markets and suitability of a design's power output.

Most of the designs developed so far have aimed at increased safety and economical operation by simplifying construction, operation and maintenance because these designs have included these factors as principal objectives.

The Advanced Boiling Water Reactor (ABWR) and its smaller counterpart the Simplified Boiling Water Reactor (SBWR) are among the newer designs in BWR proposed by General Electric (GE) (Reference New Reactor[6]). The ABWR and SBWR designs are expected to show improvement over the current fleet of BWRs in plant availability, operating capacity factor, safety and reliability while reducing power generation costs, construction times, occupational radiation exposure and radioactive waste. By simplifying design of the components system and structure and by using natural circulation of coolant further improvements in safety, performance and economy can be made.

The designs of ABWR (fig 1.1) and SBWR (fig 1.2) have much in common, the only difference between them are the power rating, core flow recirculation (10 internal pumps for ABWR and natural circulation) and extent to which some of safety systems uses active versus passive features. An important characteristic of both ABWR and SBWR is the elimination of external circulation piping and therefore permits a compact containment design. It also allows elimination of large vessel nozzles below the core, and therefore the design of a more economic emergency core cooling systems. Elimination of external circulation piping results in a greater then 50 % reduction in welds, less in service inspection (ISI) of the primary system boundary, less occupational dose during ISI, increased system integrity and sizable saving in capital cost.

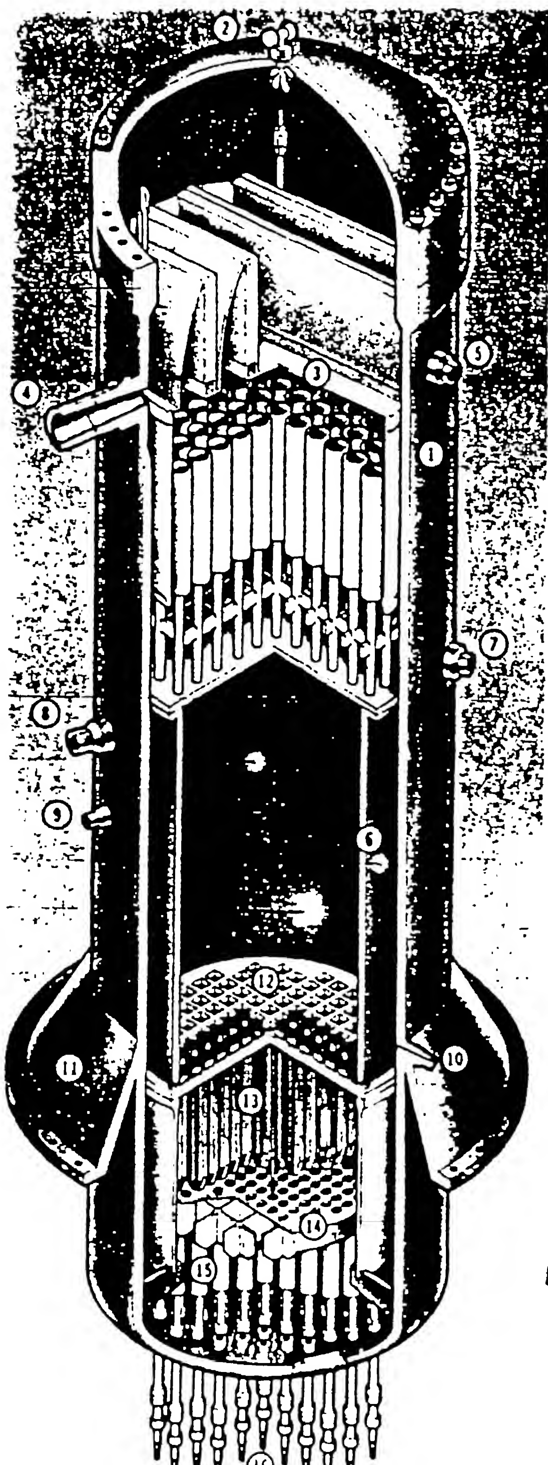
In response to the increasing interest in potential future nuclear units comprising the characteristics of smaller size, greater simplicity and more passive safety features, GE started



Advanced Boiling Water Reactor Assembly

1 Vent and Head Spray
2 Steam Dryer
3 Steam Outlet Flow Restrictor
4 Steam Separators
5 RPV Stabilizer
6 Feedwater Sparger
7 Shutdown Cooling Outlet
8 Low Pressure Flooder (LPFL) and Shutdown Cooling Sparger
9 High Pressure Core Flooder (HPCF) Sparger
10 HPCF Coupling
11 Top Guide
12 Fuel Assemblies
13 Core Shroud
14 Control Rod
15 Core Plate
16 In-Core Instrument Guide Tubes
17 Control Rod Guide Tubes
18 Core Differential Pressure Line
19 Reactor Internal Pumps (RIP)
20 Thermal Insulation
21 Control Rod Drive Housings
22 Fine Motion Control Rod Drives
23 RIP Motor Casing
24 Local Power Range Monitor

Fig. (1.1) Advanced Boiling Water Reactor Assembly.



Simplified Boiling Water Reactor Assembly

1 Reactor Pressure Vessel
2 RPV Top Head
3 Integral Dryer-Separator Assembly
4 Main Steam Line Nozzle
5 Depressurization Valve Nozzle
6 Chimney
7 Feedwater Inlet Nozzle
8 Reactor Water Cleanup/Shutdown Cooling Suction Nozzle
9 Isolation Condenser Return Nozzle
10 Gravity-Driven Cooling System Inlet Nozzle
11 RPV Support Skirt
12 Core Top Guide Plate
13 Fuel Assemblies
14 Core Plate
15 Control Rod Guide Tubes
16 Fine Motion Control Rod Drives

Fig.(1.2) Simplified Boiling Water Reactor Assembly.

(Drawing courtesy of GE)

studies in 1982 of a 600 MWe BWR with simplified power generation, safety and heat removal systems. The basic objectives that were established for this new design SBWR are

- Power generation cost to be cheaper than those of coal.
- Plant safety design simpler than in current design by using
- passive safety concepts.
- Design based on existing technology.
- Shorter construction schedules.
- Electrical rating in the 600 MWe range.
- Improve the seismic resistance of the core.
- Lengthen the continuous operating period.
- Simplify the system by eliminating movable components.
- Improve the operability and the maintainability.

Selection of the natural circulation as the means for providing coolant flow through the reactor, coupled with a 42 Kw/l core power density, results in a number of benefits to help satisfying SBWR objectives, Compared to the existing, forced circulation plants, the natural circulation BWR offers low fuel cycle costs, fewer operational transients, and increased thermal margin for the transients expected to occur. In addition, elimination of the recirculation, pumps and controls needed for forced circulation substantially simplified the design. (fig 1.3)

Conventional BWR safety / relief valves - which opens and closes to discharge reactor vessel steam to suppression pool - are not needed in SBWR because in it an isolation condenser is placed in the isolation condenser pool so when the reactor vessel is isolated from the turbine condenser, the isolation condenser controls the reactor pressure automatically without the need to

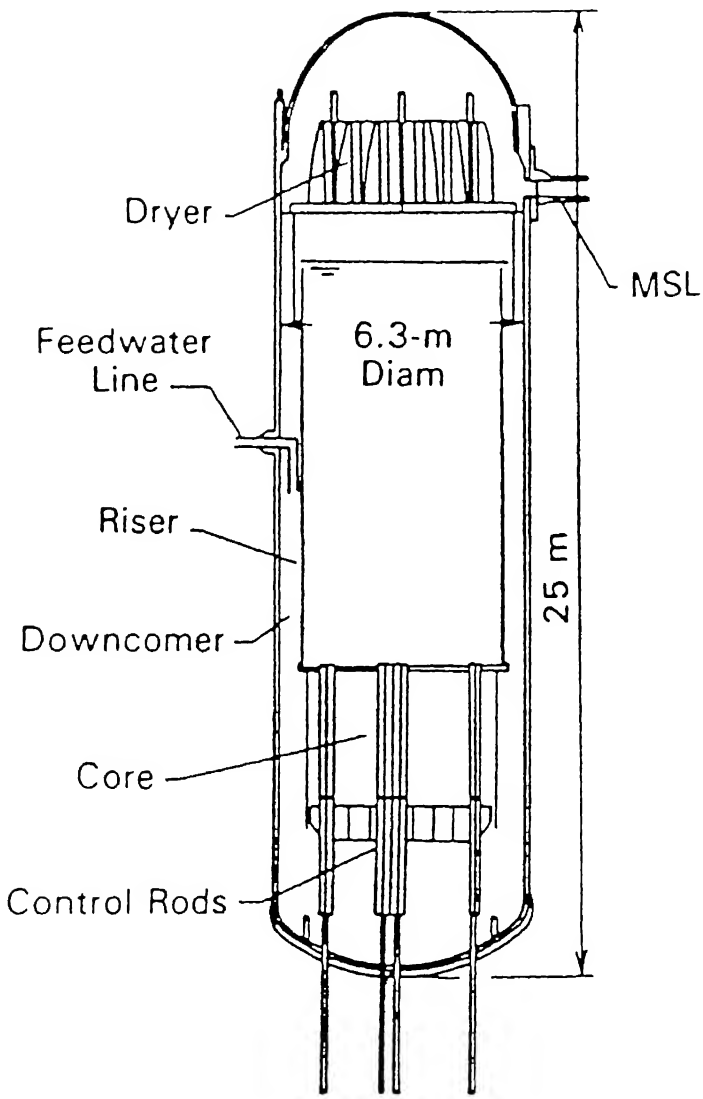


Fig.(1.3) Schematic of Reactor Pressure Vessel of SBWR.

remove fluid from the reactor vessel.

Gravity-driven core cooling system provide a simple approach to emergency core cooling system because it eliminates the need of pumps. Simplification of the components and the system made the primary containment vessel and reactor building smaller which reduces the radiation exposure and construction cost substantially.

CHAPTER 2

PRINCIPLE OF NUCLEAR REACTORS

2.1 Introduction:

The thermal energy produced in a fission power plant is the kinetic energy of the fission fragments and to a lesser extent of the emitted neutron and other particles and radiation such as gamma rays which get converted to heat when these particles are absorbed. This heat is removed by a coolant and subsequently utilized in a thermodynamic cycle.

Nuclear reactors are variously classified according to general purpose or function, type of the moderator, type of the coolant, neutron energy classification, type of fuel, type of core internal design and other factors.

While the other possible coolant besides light water can be used like the heavy water, organic liquid, gases and liquid metals, light water has been used most extensively , because of its availability, low pumping power and the advanced state of knowledge concerning its chemistry and thermodynamics properties etc.

2.2 Light Water Reactors:

The reactor in which the coolant is light water may be broadly divided into PWR and BWR. As the names suggest, Boiling Water Reactors are those in which the water boils within the reactor, and in Pressurised Water Reactor the pressure is high to avoid boiling. Thus the water pressure corresponds to the saturation temperature at the reactor pressure in a BWR. The reactor pressure is roughly between 600 and 1000 psia. A schematic

arrangement of a BWR is shown in the figure(2.1)

In a Pressurised Water Reactor, reactor pressures are higher and of the order of 2000 psia, for the primary coolant loop within the reactor. A heat exchanger in which the primary coolant exchanges heat with a secondary coolant is used. A schematic sketch of a PWR is shown in the figure(2.2)

However, in this study, we focus our attention on the former type of reactor that is the BWR. The boiling reactor has a function closely resembling that of boiler in a conventional fossil-fuel steam power plant and is basically simpler than it. While in a boiler heat is transmitted from the furnace to the water indirectly - partly by radiation, partly by convection and partly by conduction; with combustion gases acting as an intermediate agent or coolant, in the Boiling Water Reactor, the coolant is in direct contact with the heat producing nuclear fuel and boils in the same compartment in which the fuel is located.

The simplest form of a boiling reactor power plant as shown in the figure(2.3)consists of a reactor, a turbine generator, a condenser and the associated equipments (such as ejector, cooling system etc.) and a feed pump to force the incoming coolant to core. Slightly sub-cooled liquid enters the reactor core in the bottom where it received the sensible heat for saturation plus latent heat for vaporisation. When it reaches the top of the core it has been converted to a very wet mixture of liquid and vapor. The vapor separates from the liquid, flows to the turbine, does work and is condensed by the condenser, and the condensate is then pumped back to the reactor by the feed pump. That is how the power is generated in the conventional BWR. Such a cycle is called

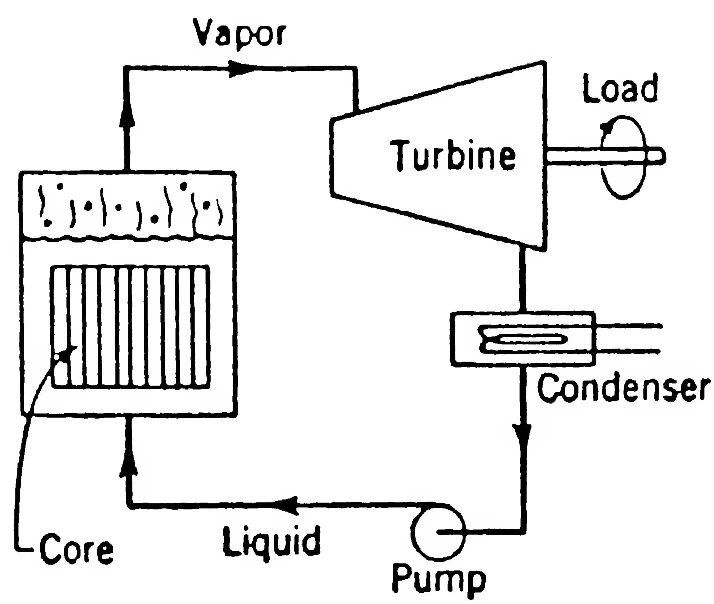


Fig 2·1 Schematic arrangement of boiling-reactor power plant.

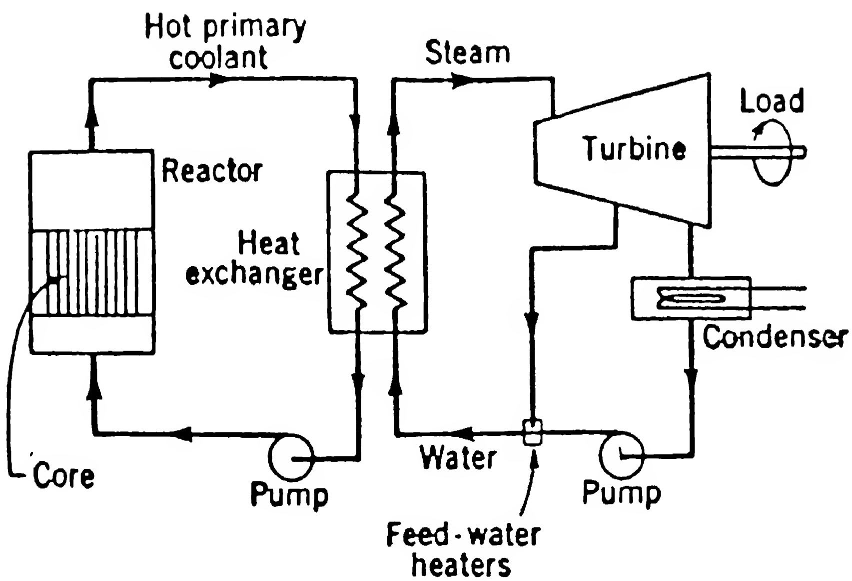


Fig 2·2 Schematic arrangement of liquid-cooled-reactor power plant.

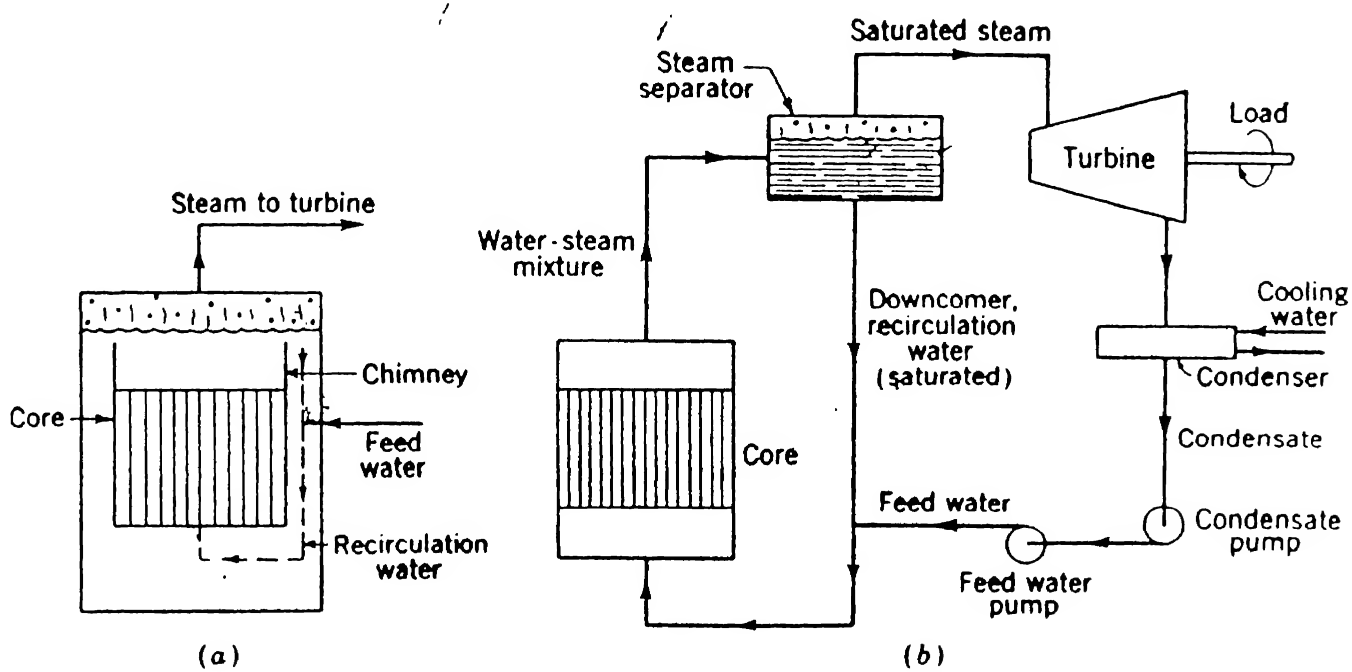


Fig. 2.3 Simple boiling-water-reactor systems. (*a*) Internal recirculation; (*b*) external recirculation.

direct cycle BWR, which has the disadvantage of primary coolant coming in contact with turbine etc. To overcome this problem one can employ indirect cycle BWR in which a heat exchanger is used to separate the radioactive primary coolant from the non-radioactive secondary coolant.

2.3 Natural Circulation BWR:

The simplified and modified version of the conventional BWR is the natural circulation BWR, more generally known as Simplified Boiling Water Reactor (SBWR). The present thesis deals with this simplified BWR. Except for the neutronic / thermal-hydraulic design, there is no difference between the design of natural circulation BWR and forced circulation BWR. In the natural circulation BWR the necessary recirculation flow fore core cooling will be produced by the driving force originating from the buoyancy due to density head difference in the closed loop of the recirculation flow. (in the absence of recirculation pumps)

Actually the technology for the natural circulation is not new to BWR. The Dodewaard plant in Netherlands has operated on this principle at a life time capacity factor of 84%. Larger BWRs have been operated at 50% power levels in natural circulation mode to prove that BWR of this type is indeed possible.

A schematic of the natural circulation BWR as it contrasts with the forced circulation BWR is shown in the figure(2.4). The major design aspects of the natural circulation BWR has been shown in the figure (2.5).

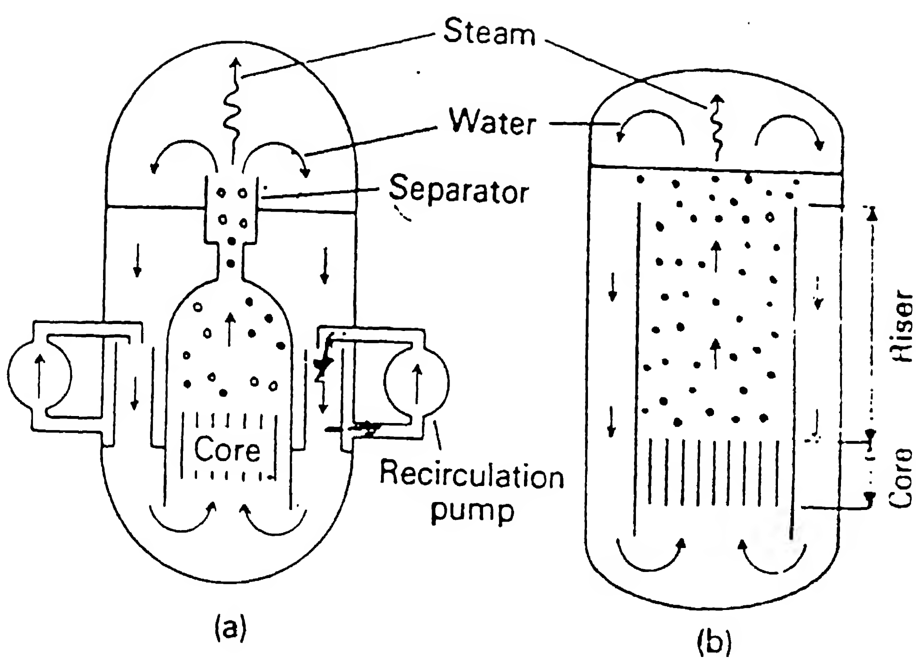


Fig.2.4 Schematic of (a) forced-circulation BWR and (b) natural-circulation BWR.

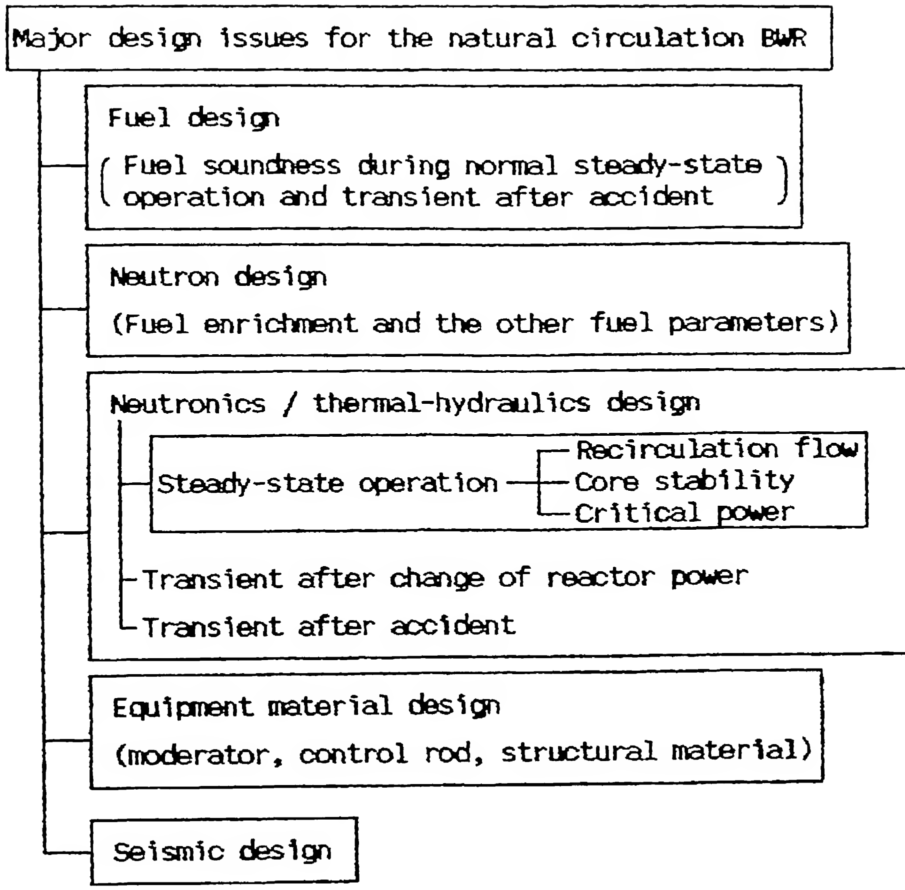


Figure (2.5): Major design aspects of natural circulation BWR

CHAPTER 3

PROBLEM STATEMENT AND PROCEDURE

3.1 Problem Statement:

The present study investigate the feasible power rates for a natural circulation BWR. For this the core stability of the reactor was to be examined. It further studies effects of the variation in active fuel length and core power density on the necessary riser height for a natural circulation BWR.

3.2 Procedure :

In the analysis, a simple flow model was adopted where the main recirculation flow in the riser, downcomer and parallel core channels was assumed to be vertically one dimensional.

A code was developed to analyze the nuclear-coupled stability as well as thermal-hydraulic stability of the fuel channels. This stability code consists of a reactor point kinetics model and an axially one dimensional thermal hydraulic model of the fuel channels, and it adopts approximately the same line of approach as used in Lahey & Moody [7]. This stability code calculates the frequency response of the open loop of the system and obtains the decay ratio. The decay ratio is defined in chapter(4) equation (123). The input parameters for this code are neutron data as well as the thermal hydraulic data and the channel flow.

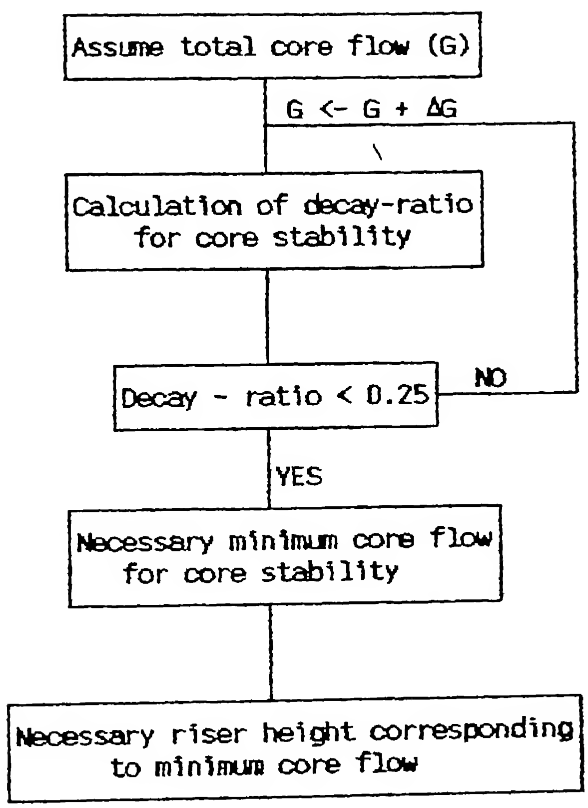
Finally by balancing the total pressure drops in the respective parts of the recirculation flow loop formed by the down comer-core-riser with the total driving pressure, the necessary riser height is calculated.

figure(3.1) shows the procedure for calculating the

necessary riser height. First we obtain the minimum recirculation flow rate necessary to satisfy a tentative requirement on the decay-ratio (DR) as

$$DR < 0.25$$

This requirement for decay ratio is conservative enough to give an overall inherently safe reactor design. Finally based on this minimum recirculation flow rate necessary riser height is calculated.



Figure(3.1): Calculation procedure for necessary riser height.

CHAPTER 4

THEORETICAL ANALYSIS

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4.1 Types of Instabilities in BWR:

4.1.1 Nuclear coupled instability:

During the early days of BWR technology, there was considerable concern about nuclear-coupled instability ; that is interaction between the random boiling process and void-reactivity feedback modes. Indeed, it has been proved by extensive series of experiments that while instability was observed at lower pressures but it was not expected to be a problem at the higher system pressures. In natural circulation BWR there is some probability of arriving at low reactor pressure hence nuclear coupled instability analysis become a necessary for this reactor.

In addition to nuclear coupled instability, there are number of static and dynamic instabilities. Some of the static and dynamic instabilities which are generally considered in design of BWR, are

4.1.2 Flow Excursion Instability :

It is concerned with the interaction between the pump's head-flow characteristics and the hydraulics characteristics of the boiling channel. Since in natural circulation pumps are eliminated hence this instability does not occur.

4.1.3 Flow Regime Relaxation Instability :

It is due to the change in the flow regimes. This instability is self stable in case of natural circulation BWR because if due to a small perturbation a slug flow pattern changes to annular flow a lower pressure drop is experienced which may tend to increase the flow rate and thus cause the flow regime to return to

its previous state.

4.1.4 Density Wave Oscillations :

This instability is due to the feedback and interaction of various pressure drop components and is caused specially by the lag introduced through the density head term due to the finite speed of propagation of kinematic density wave. This instability is obviously quite important and needs to be examined.

4.1.5 Pressure Drop Oscillation :

It is the instability mode in which flow excursion instability and a compressible volume in the boiling system interact to produce a fairly low frequency oscillations. This instability is normally not a problem in natural circulation BWR.

There are however, other instability modes that can occur in two phase systems. Since here we are concerned exclusively with natural circulation BWR technology, we can neglect them here.

4.2 Stability Analysis:

4.2.1 Principle:

Consider a subcooled flow entering a heated channel as shown in figure (4.1). The boiling boundary, $\lambda(t)$, defined as the instantaneous location of the point where the bulk fluid temperature reaches saturation temperature, oscillates due to the inlet flow and power density fluctuations. Change in flow and the length of the single phase region combine to create an oscillatory single phase pressure drop. At the boiling boundary these perturbations are transformed into quality (or void fraction) perturbations that travel up the heated channel with the flow. Combined effects of flow and void fraction perturbations and the variation in the two phase length create a two phase pressure drop perturbation. However the total pressure drop across the boiling

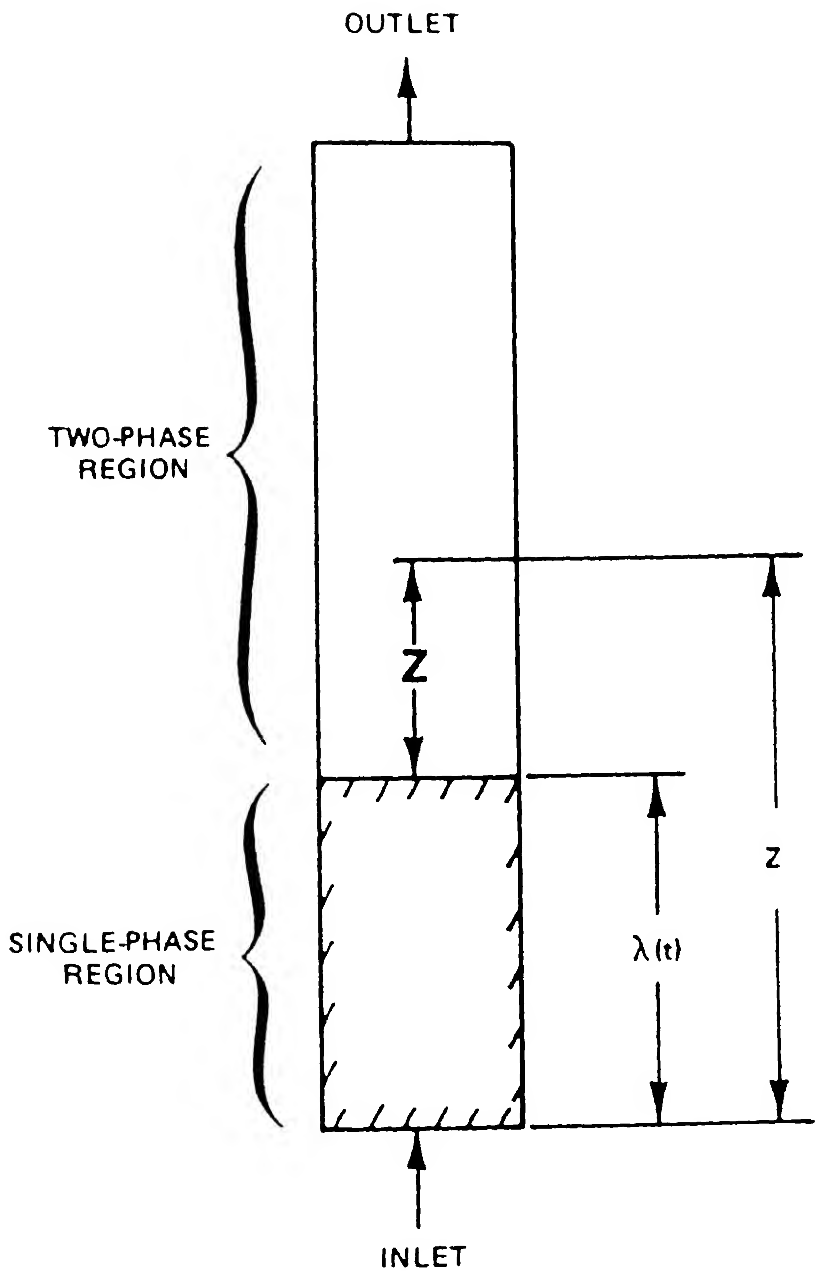


Fig. 4.1 Schematic of a heated section.

channel has to remain constant with time in BWR, thus, the two phase pressure drop perturbation produces a feedback perturbation of the apposite sign in the single phase region.

To simplify the analysis and yet retain the essential features, the following assumptions are considered in the present stability analysis of natural circulation BWR.

- Diabatic homogeneous two phase flow.
- Constant system pressure.
- Uniform axial heat flux.
- No subcooled boiling.

The basic solution scheme consists of coupling analytical models for the single and two phase region with a model for the dynamic thermal response of the heated surface, such that imposed boundary conditions are satisfied.

4.2.2 Model For The Fuel-rod:

The present analysis for fuel rod essentially follows the treatise given in Lahey & Moody [7].

Using a single-node lumped heat capacity model one can write-

$$M\cdot C_f\cdot\frac{dT_f}{dt} = q\cdot V - q''\cdot A \qquad \text{-------} \qquad (1)$$

where

M = Mass of the fuel rod.

C_f = Specific heat of the fuel rod.

T_f = Fuel temperature.

q = Fuel power density.

q'' = Heat flux.

V = Volume of fuel rod.

A = Surface area of fuel rod.

At steady state

$$q_0 \cdot V = q_0'' \cdot A \qquad (2)$$

where

q_0 = Steady state power density.

q_0'' = Steady state heat flux.

Let

$$Q = \frac{q - q_0}{q_0} \qquad (3)$$

$$Q'' = \frac{q'' - q_0''}{q_0''} \qquad \text{----------} \quad (4)$$

$$\Pi_f = T_f - T_{f0} \qquad \text{-----------} \quad (5)$$

$$\Pi_c = T_c - T_{c0} \qquad \text{-----------} \quad (6)$$

where

T_{f0} = Steady state fuel temperature.

T_c = Coolant temperature.

T_{c0} = Steady state coolant temperature.

Eliminating q, q" and T_f from (1) by using (3),(4) and (5),

$$M \cdot C_f \cdot \frac{d\Pi_f}{dt} = Q \cdot V \cdot q_0 - Q'' \cdot A \cdot q_0'' \qquad \text{--------} \quad (7)$$

Now taking Laplace Transformation of above equation,

$$M \cdot C_f \cdot s \cdot \bar{\Pi}_f = q_0 \cdot V \cdot \bar{Q} - q_0'' \cdot A \cdot \bar{Q}'' \qquad \text{---------} \quad (8)$$

Now to express perturbation in heated wall temperature ($\bar{\Pi}_f$) in terms of heat flux perturbation ($\bar{Q}''$) use -

$$q'' = h \cdot (T_f - T_c) \qquad \text{-------} \quad (9)$$

$$q_0'' = h_0 \cdot (T_{f0} - T_{c0}) \qquad \text{-------} \quad (10)$$

where

h = Heat transfer coefficient of coolant.

h_0 = Steady state heat transfer coefficient of coolant.

From (4),(9) and (10) and after simplifying,

$$Q'' = \frac{h - h_0}{h_0} + \frac{h}{q''_0} \cdot (\Pi_f - \Pi_c) \quad \text{--------} \quad (11)$$

The second term in right hand side in the above equation is non-linear it can be linearized by replacing h by h_0 in that term hence

$$Q'' = H + \frac{h_0}{q''_0} \cdot (\Pi_f - \Pi_c) \quad \text{---------} \quad (12)$$

here

$$H = \frac{h - h_0}{h_0} \quad \text{-------} \quad (13)$$

Now from Reynold's Analogy

$$h = 0.23 \cdot \frac{k}{L} \cdot Re^{0.8} \cdot Pr^{0.3} \quad \text{-------} \quad (14)$$

where

k = Thermal conductivity of coolant.

L = Length of fuel rod.

Re = Reynold's Number

= constant$\cdot$G

G = Coolant flow rate

Pr = Prandtl Number

Since k,L and Pr are constant for given coolant and operating pressure of system hence

$$h = \text{constant} \cdot G^{0.8} \quad \text{--------} \quad (15)$$

and $$h_0 = \text{constant} \cdot G_0^{0.8} \quad \text{--------} \quad (16)$$

where G_0 = Steady state coolant flow rate.

Differentiating (15) with respect to G,

$$\frac{dh}{dG} = \text{constant} \cdot 0.8 \cdot G^{-0.2} \quad \text{-------} \quad (17)$$

Combining (16) and (17) ,

$$\frac{dh}{h_0} = 0.8 \cdot \frac{dG \cdot G^{-0.2}}{G_0^{0.8}} \quad \text{--------} \quad (18)$$

To linearize equation (18) replace G by G_0 ,

$$\frac{dh}{h_0} = 0.8 \cdot \frac{dG}{G_0} \quad \text{-------} \quad (19)$$

or $$H = 0.8 \cdot \mathbb{G} \quad \text{-------} \quad (20)$$

here $\mathbb{G}$ = non-dimensional perturbation in coolant flow rate

Substituting H from (20) into (12) and taking Laplace Transformation of the resulting equation one gets

$$\bar{Q}'' = 0.8 \cdot \bar{\mathbb{G}} + \frac{h_0}{q''_0} \cdot (\bar{\Pi}_f - \bar{\Pi}_c) \quad \text{-------} \quad (21)$$

Now eliminating $\bar{\Pi}_f$ from (8) and (21) and simplifying using (2) it reduces to -

$$\bar{Q}'' = \left(\frac{M \cdot C_f \cdot s}{M \cdot C_f \cdot s + h_0 \cdot A} \right) \cdot \left[0.8 \cdot \bar{\mathbb{G}} + \frac{h_0 \cdot A}{M \cdot C_f \cdot s} \bar{\mathbb{Q}} - \frac{h_0}{q''_0} \bar{\Pi}_c \right] \text{--} \quad (22)$$

Equation (22) gives the response of the heated wall in the single phase region to perturbations in coolant flow, core power density and coolant temperature.

4.2.3 Model for The Single Phase Region:

The analysis for the single phase region essentially follows same line of approach as used in Lahey & Moody [7].

Single phase energy balance can be written as

$$A_c \cdot \rho_c \cdot C_c \cdot \frac{\partial T_c}{\partial t} + G \cdot C_c \cdot \frac{\partial T_c}{\partial z} = q'' \cdot A_p \quad \text{--------} \quad (23)$$

where

A_c = Cross-section area for coolant flow.

ρ_c = Coolant density.

C_c = Specific heat of coolant.

A_p = Perimeter of fuel rod.

t and z are time and distance (from core inlet) variables.

At the steady state equation(23) reduces to,

$$\left(\frac{\partial T_c}{\partial z}\right)_0 = \frac{q''_0 \cdot A_p}{G_0 \cdot C_c} \text{ ------ } (24)$$

Now substituting $T_c = \Pi_c + T_{c0}$ in (23) and then simplifying ,

$$A_c \cdot \rho_c \cdot C_c \cdot \frac{\partial \Pi_c}{\partial t} + G \cdot C_c \cdot \frac{\partial \Pi_c}{\partial z} + G \cdot C_c \cdot \left(\frac{\partial T_c}{\partial z}\right) = q'' \cdot A_p \text{ --- } (25)$$

Making the above equation linearize and subtracting $q''_0 \cdot A_p$ from both sides of equation (25) ,

$$A_c \cdot \rho_c \cdot C_c \cdot \frac{\partial \Pi_c}{\partial t} + G \cdot C_c \cdot \frac{\partial \Pi_c}{\partial z} + G \cdot C_c \cdot \left(\frac{\partial T_c}{\partial z}\right)_0 - q''_0 \cdot A_p = (q'' - q''_0) \cdot A_p$$

$$\text{--------- } (26)$$

Simplifying (26) using (24) and transforming q" and G by Q" and $\mathbb{G}$,

$$\frac{A_c \cdot \rho_c}{G_0} \cdot \frac{\partial \Pi_c}{\partial t} + \frac{\partial \Pi_c}{\partial z} = \frac{q''_0 \cdot A_p}{G_0 \cdot C_c} \cdot (\mathbb{Q} - \mathbb{G}) \text{ ------- } (27)$$

Now taking Laplace Transform of above equation,

$$\frac{A_c \cdot \rho_c}{G_0} \cdot s \cdot \bar{\Pi}_c + \frac{d\bar{\Pi}_c}{dz} = \frac{q''_0 \cdot A_p}{G_0 \cdot C_c} \cdot (\bar{\mathbb{Q}} - \bar{\mathbb{G}}) \text{ ------- } (28)$$

Now substituting $\bar{Q}''$ from (22) in above equation,

$$X(s) \cdot \bar{\Pi}_c + \frac{d\bar{\Pi}_c}{dz} = Y(s) \cdot \bar{\mathbb{G}} + Z(s) \cdot \bar{\mathbb{Q}} \text{ -------- } (29)$$

where

$$X(s) = \frac{A_c \cdot \rho_c}{G_0} \cdot s + \frac{A_p \cdot M \cdot C_f \cdot h_0}{G_0 \cdot C_c} \cdot \left[\frac{s}{M \cdot C_f \cdot s + h_0 \cdot A}\right] \text{ ---- } (30)$$

$$Y(s) = \frac{q''_0 \cdot A_p}{G_0 \cdot C_c} \cdot \left[\frac{0.8 \cdot M \cdot C_f \cdot s}{M \cdot C_f \cdot s + h_0 \cdot A} - 1\right] \text{ -------- } (31)$$

$$Z(s) = \frac{q''_0 \cdot A_p}{} \cdot \left[\frac{h_0 \cdot A}{}\right] \text{ --------- } (32)$$

Equation (29) is linear differential equation of first order hence can be solved easily, solving it with the assumption that perturbation in inlet coolant temperature is zero, one gets

$$\bar{\Pi}_c = [\ Y(s)\cdot\bar{G}' + Z(s)\cdot\bar{Q}\ ]\cdot z\cdot \left[\frac{1 - e^{-X(s)\cdot z}}{X(s)\cdot z}\right] \quad ------ (33)$$

Next step in the analysis is to determine the dynamics of the boiling boundary. For the assumption of uniform axial heat flux, the coolant enthalpy increases linearly with axial distance from the bottom of fuel rod. In fact a positive perturbation in coolant temperature at the boiling boundary causes a negative perturbation in the boiling boundary as shown in fig.(4.2)

Let λ = Non-boiling height

λ_0 = Steady state non-boiling height

λ' = Perturbation in non-boiling height

Now making energy balance in the length λ' at boiling boundary

$$\lambda'\cdot A_p\cdot q_0'' = -\ G_0\cdot C_c\cdot \Pi_c(\ \lambda_0) \quad -------- (34)$$

Taking Laplace Transform of above equation ,

$$\bar{\lambda}' = -\ \frac{G_0\cdot C_c}{q_0''\cdot A_p}\cdot \bar{\Pi}_c(\ \lambda_0) \quad -------- (35)$$

Now substituting $\bar{\Pi}_c(\ \lambda_0)$ from (33) into (35) one gets

$$\bar{\lambda}' = -\ \frac{G_0\cdot C_c}{q_0''\cdot A_p}\cdot [\ Y(s)\cdot\bar{G} + Z(s)\cdot\bar{Q}]\cdot\lambda_0\cdot \left[\frac{1 - \exp(\ X(s)\cdot\lambda_0)}{X(s)\cdot\lambda_0}\right]$$

$$---------- (36)$$

This can be written as

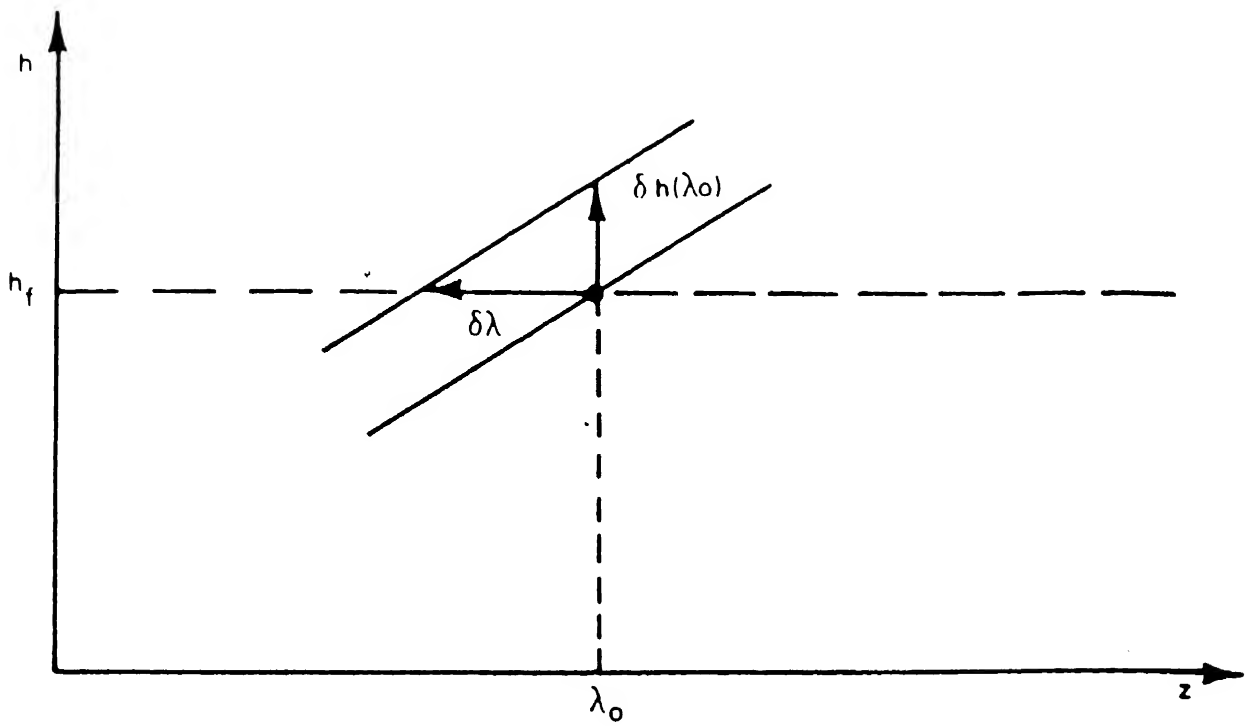


Fig. 4.2 Boiling boundary perturbations.

$$\bar{\lambda}' = \Lambda_1(s)\cdot\bar{G}' + \Lambda_2(s)\cdot\bar{Q} \qquad \text{--------- (37)}$$

where

$$\Lambda_1(s) = -\frac{G_0\cdot C_c}{q_0''\cdot A_p}\cdot\left[\frac{1-\exp(X(s)\cdot\lambda_0)}{X(s)\cdot\lambda_0}\right]\cdot Y(s) \qquad \text{----- (38)}$$

and

$$\Lambda_2(s) = -\frac{G_0\cdot C_c}{q_0''\cdot A_p}\cdot\left[\frac{1-\exp(X(s)\cdot\lambda_0)}{X(s)\cdot\lambda_0}\right]\cdot Z(s) \qquad \text{----- (39)}$$

The main purpose of the single phase analysis has been to derive the expression for the dynamics of the boiling boundary for which equation (37) is the required result.

4.2.4 Model for Two-Phase Region:

The analysis for two phase region is similar to as done in Lahey & Moody [7].

For constant system pressure, the continuity equation for a homogeneous two phase mixture can be written in the form

$$\frac{\partial v}{\partial z} = \frac{q_0''\cdot A_p\cdot v_{fg}}{h_{fg}\cdot A_x} = \Omega \qquad \text{------- (40)}$$

The parameter , Ω ,which has the units of reciprocal of time, is frequently referred as the characteristic frequency of phase change. It physically represent the speed at which phase change takes place.

here

v = coolant flow velocity.

v_{fg} = change in specific volume of coolant at phase change.

h_{fg} = enthalpy change of coolant at phase change.

A_x = cross section area of coolant flow.

Corresponding energy equation for a homogeneous two phase system is given by

$$\frac{Dx}{Dt} - \Omega\cdot x = \Omega\cdot\frac{v_f}{v_{fg}} \qquad \text{------- (41)}$$

here

x = quality of coolant

v_f = specific volume of coolant in liquid phase.

Equation (40) can be integrated along constant time characteristic to yield,

$$v(z,t) = v_1(t) + \int_{\lambda(t)}^{z} \Omega\cdot dz' \qquad \text{------- (42)}$$

here

$v_1(t)$ = single phase inlet flow velocity at time t.

$\lambda(t)$ is the instantaneous position of the boiling boundary which can be written as

$$\lambda(t) = \int_{t-\nu}^{t} v_1(t')\cdot dt' \qquad \text{-------- (43)}$$

The parameter ν is the time required for a fluid particle to lose its sub-cooling. It can be evaluated by making energy balance in the sub-cooled section, i.e.

$$q''\cdot A_p\cdot L_s\cdot\nu = (\rho_c\cdot A_x\cdot L_s)\cdot(h_f - h_i) \qquad \text{------- (44)}$$

here

L_s = Length of sub-cooled section.

h_i = Enthalpy of coolant at core entrance.

hence

$$\nu = \frac{\rho_c\cdot A_x\cdot(h_f - h_i)}{q''\cdot A_p} \qquad \text{---------- (45)}$$

Neglecting acoustic phenomena one can take

$$v(z,t) = \frac{dz}{dt} \qquad \text{-------- (46)}$$

Equation (46) can be used in conjunction with (42) to yield,

$$\frac{dz}{dt} = v_1(t) + \int_{\lambda(t)}^{z} \Omega \cdot dz' \quad \text{--------} \quad (47)$$

or

$$\frac{dz}{dt} = v_1(t) + \Omega \cdot (z - \lambda) \quad \text{-------} \quad (48)$$

$$\Rightarrow \quad v_1(t) = \frac{dz}{dt} - \Omega \cdot (z - \lambda) \quad \text{--------} \quad (49)$$

The next task is to integrate eq^{n} (49) to obtain the locus of the fluid particles in space-time plane. It is convenient to transform from the axial coordinate, z , referred to the core inlet , to y, which is referred to the boiling boundary.

Thus

$$y = z - \lambda(t) \quad \text{--------} \quad (50)$$

Differentiate (50) and (43) with respect to t ,

$$\frac{dy}{dt} = \frac{dz}{dt} - \frac{d\lambda(t)}{dt} \quad \text{--------} \quad (51)$$

and

$$\frac{d\lambda(t)}{dt} = v_1(t) - v_1(t - \nu) \quad \text{------} \quad (52)$$

Now combining (49),(50),(51) and (52) one obtains

$$\frac{dy}{dt} - \Omega \cdot y = v_1(t - \nu) \quad \text{--------} \quad (53)$$

This is a linear differential equation of first order hence can be solved as

$$y(t) = \exp(\Omega \cdot t) \cdot \int_{t_0}^{t} \exp(-\Omega \cdot t') \cdot v_1(t' - \nu) \cdot dt' \quad \text{--------} \quad (54)$$

Here t_0 is the time at which fluid particle crosses the boiling boundary.

Let

$$\left.\begin{aligned} \tau &= t - t_0 \\ \tau_1 &= t' - t_0 \\ d\tau_1 &= dt' \end{aligned}\right] \quad \text{------} \quad (55)$$

using (55) in (54) one gets

$$y(\tau) = \exp(\Omega \cdot \tau) \cdot \int_{0}^{\tau} \exp(-\Omega \cdot \tau_1) \cdot v_1(\tau_1 - \nu) \cdot d\tau_1 \quad \text{--------} \quad (56)$$

Equation (56) can be perturbated with respect to v_1 and τ and

Laplace Transformed to yield,

$$\frac{\partial \bar{y}}{\partial v_1} = \frac{\exp(-s\cdot\nu)\cdot[\ 1 - \exp\{(\Omega - s)\cdot\tau\}]}{(s - \Omega)} \quad \text{-------} \ (57)$$

and $$\frac{\partial \bar{y}}{\partial \tau} = v_{10}\cdot \exp(\Omega\cdot\tau) \quad \text{--------} \ (58)$$

here

v_{10} = Steady state single phase coolant flow velocity.

Let

τ' = perturbation in τ

and v_1' = perturbation in v_1

Now $\bar{\tau}'$ must be expressed in terms of $\bar{\mathbb{G}}$ and $\bar{\mathbb{Q}}$, for this perturbate and Laplace transform equation (50) at a fixed axial position ,z , then

$$dz = 0 = \frac{\partial \bar{y}}{\partial v_1}\cdot \bar{v}_1' + \frac{\partial \bar{y}}{\partial \tau}\cdot \bar{\tau}' + \bar{\lambda}' \quad \text{--------} \ (59)$$

By combining (57),(58) and (59) one obtains

$$\bar{\tau}' = -\frac{e^{-\Omega\cdot\tau}}{v_{10}}\cdot\left[\ \bar{\lambda}' + \frac{e^{-s\cdot\nu}}{(s-\Omega)}\cdot\left(1 - \exp\{(\Omega - s)\cdot\tau\}\right)\cdot \bar{v}_1'\ \right] \text{--} \ (60)$$

But

$$\left.\begin{aligned} & G = v_1\cdot A_x\cdot\rho_c \\ \text{so}\quad & \mathbb{G} = \frac{dG}{G_0} = \frac{dv_1}{v_{10}} \\ \text{hence}\quad & \bar{v}_1' = v_{10}\cdot \bar{\mathbb{G}} \end{aligned}\right] \quad \text{------} \ (61)$$

Now substituting $\bar{v}_1'$ from (61) into (60) ,

$$\bar{\tau}' = -\frac{e^{-\Omega\cdot\tau}}{v_{10}}\cdot\left[\ \bar{\lambda}' + \frac{e^{-s\cdot\nu}}{(s-\Omega)}\cdot\left(1 - \exp\{(\Omega - s)\cdot\tau\}\right)\cdot v_{10}\cdot \bar{\mathbb{G}}\ \right] \quad \text{-------} \ (62)$$

To relate perturbation in coolant quality with perturbation in τ one proceeds as follows-

Solving equation (41), which is a linear differential equation of first order

$$x(t) = e^{\Omega\cdot t} \cdot \int_{t_0}^{t} \Omega\cdot \frac{v_f}{v_{fg}} \cdot e^{-\Omega\cdot t'} \cdot dt' \quad \text{-------} \ (63)$$

Using (55) the equation (63) can be integrated and simplified as

$$x(\tau) = \frac{v_f}{v_{fg}} \cdot (e^{\Omega\cdot\tau} - 1) \quad \text{---------} \ (64)$$

Now by perturbating and Laplace Transforming equation (64) one gets

$$\bar{x}' = \frac{v_f}{v_{fg}} \cdot e^{\Omega\cdot\tau} \cdot \bar{\tau}' \quad \text{---------} \ (65)$$

here x' is the perturbation in the coolant quality.

In the BWR, the nuclear feedback is largely through void reactivity coupling, thus ,it is important to relate quality perturbations to perturbation in void fraction. A reasonably accurate void-quality model has given the relation between void fraction and the quality as-

$$\alpha = \frac{x}{C_0\cdot \left[x + \frac{\rho_g}{\rho_f}\cdot(1 - x) \right]} \quad \text{-------} \ (66)$$

where

α = void fraction

ρ_g = coolant density in vapor phase

ρ_f = coolant density in liquid phase

and C_0 = $[(0.71 + 0.0001\cdot p) - 1]^{-1}$

here p = pressure of reactor (in psia)

By perturbating and Laplace Transforming equation (66) one gets

$$\bar{\alpha}' = K \cdot \bar{x}' \quad \text{--------} \ (67)$$

where

$$K = \frac{\rho_g}{C_0\cdot\rho_f\cdot\left[x_0 + \frac{\rho_g}{\rho_f}\cdot(1 - x_0) \right]^2} \quad \text{-------} \ (68)$$

here

α' = perturbation in void fraction

x_0 = steady state coolant quality

By combining (62),(65) and (68) one gets

$$\bar{\alpha}' = -K\cdot\Omega\cdot\frac{v_f}{v_g}\cdot\left[\frac{\bar{\lambda}'}{v_{10}} + \frac{e^{-s\cdot\nu}}{(s-\Omega)}\cdot[1-\exp\{(\Omega-s)\cdot\tau\}]\cdot\bar{G}\right] \quad \text{--- (69)}$$

Now to get perturbation in the core-average void-fraction defined as

$$\begin{aligned}
\bar{\alpha}'_{avg} &= \frac{1}{L}\cdot\int_{\lambda_0}^{L}\bar{\alpha}'\cdot dz \\
&= \frac{1}{L}\cdot\int_{0}^{(L-\lambda_0)}\bar{\alpha}'\cdot dy \\
&= \frac{1}{L}\cdot\int_{0}^{(t_{ex}-t_0)}\bar{\alpha}'\cdot\frac{dy}{dt}\cdot dt \\
&= \frac{1}{L}\cdot\int_{0}^{\tau_{ex}}\bar{\alpha}'\cdot\frac{dy}{d\tau}\cdot d\tau \\
&= \frac{1}{L}\cdot\int_{0}^{\tau_{ex}}\bar{\alpha}'\cdot v_{10}\cdot e^{\Omega\cdot\tau}\cdot d\tau \quad \text{------ (71)}
\end{aligned}$$

Here t_{ex} is the time required by fluid particle to reach at the top of the heated wall, and τ_{ex} is the time in which the fluid particle has travelled in the two phase region.

Substituting $\bar{\alpha}'$ from (70) into (71) and solving the resultant equation,

$$\bar{\alpha}' = -\frac{K\cdot v_f}{L\cdot v_{fg}}\cdot\bar{\lambda}'\cdot[\exp(\Omega\cdot\tau_{ex})-1] + B(s) \quad \text{------- (72)}$$

where

$$B(s) = -\frac{K\cdot\Omega\cdot v_f}{L\cdot v}\cdot v_{10}\cdot\frac{e^{-s\cdot\nu}}{(s-\Omega)}\cdot\left[\frac{\exp(\Omega\cdot\tau_{ex})-1}{\Omega} - \frac{\exp[(2\cdot\Omega-s)\cdot\tau_{ex}]-1}{(2\cdot\Omega-s)}\right]$$

Making first order approximation to the exponential function in expression of B(s) then we find that it will become zero. Hence after making first order approximation in (72),

$$\bar{\alpha}'_{avg} = - \frac{K \cdot v_f}{L \cdot v_{fg}} \cdot \bar{\lambda}' \cdot \Omega \cdot \tau_{ex} \qquad \text{--------} \; (74)$$

Now substituting for $\bar{\lambda}'$ from equation (37) in (74),

$$\bar{\alpha}'_{avg} = T_1(s) \cdot \bar{G} + T_2(s) \cdot \bar{Q} \qquad \text{--------} \; (75)$$

where

$$T_1(s) = - \Lambda_1(s) \cdot \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \qquad \text{---------} \; (76)$$

and

$$T_2(s) = - \Lambda_2(s) \cdot \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \qquad \text{---------} \; (77)$$

Now substituting $\Lambda_1(s)$ and $\Lambda_2(s)$ from (38) and (39) into (77) and making first order approximation to the exponential function,

$$T_1(s) = \frac{G_0 \cdot C_c}{q''_0 \cdot A_p} \cdot \lambda_0 \cdot Y(s) \cdot \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \qquad \text{---------} \; (78)$$

and

$$T_2(s) = \frac{G_0 \cdot C_c}{q''_0 \cdot A_p} \cdot \lambda_0 \cdot Z(s) \cdot \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \qquad \text{---------} \; (79)$$

Now substituting Y(s) and Z(s) from (31) and (32),

$$T_1(s) = \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \cdot \lambda_0 \cdot \left[\frac{0.8 \cdot M \cdot C_f \cdot s}{M \cdot C_f \cdot s + h_0 \cdot A} - 1 \right] \text{----} \; (80)$$

and

$$T_1(s) = \left[\frac{K \cdot v_f \cdot \Omega \cdot \tau_{ex}}{L \cdot v_{fg}} \right] \cdot \lambda_0 \cdot \left[\frac{h_0 \cdot A}{M \cdot C_f \cdot s + h_0 \cdot A} \right] \qquad \text{----} \; (81)$$

Solving equation (58),

$$\tau = \frac{1}{\Omega} \cdot \ln\left[\frac{v_{10}}{\Omega \cdot y + v_{10}} \right] \quad \text{-------} \ (82)$$

hence

$$\tau_{ex} = \frac{1}{\Omega} \cdot \ln\left[\frac{v_{10}}{\Omega \cdot (L-\lambda_0) + v_{10}} \right] \quad \text{-------} \ (83)$$

Equation (75) gives the perturbation in the void-fraction due to perturbation in coolant flow and core power density.

Now to analyse the instability due to density wave oscillation, perturbation in the pressure drop in single phase and two phase region are to be known.

4.2.5 Analysis for single phase pressure drop perturbation :

The analysis for this section essentially follows treatise given in Lahey & Moody [7].

The appropriate momentum equation for the single phase portion of the heater can be written as

$$-\frac{\partial p}{\partial z} = \rho_c \cdot \frac{\partial v_1}{\partial t} + \frac{f \cdot \rho_c \cdot v_1^2}{2 \cdot D_h} + \rho_c \cdot g \quad \text{-------} \ (84)$$

where

f = friction factor

D_h = Hydraulic mean diameter of coolant flow

g = Gravitational acceleration

In terms of the coolant flow rate equation (84) can be written as

$$-\frac{\partial p}{\partial z} = \frac{1}{A_x} \cdot \frac{dG}{dt} + \frac{f \cdot G^2}{2 \cdot D_h \cdot A_x \cdot \rho_c} + \rho_c \cdot g \quad \text{-------} \ (85)$$

Integrate (85) with respect to ,z, from ,z = 0, to ,z = λ, to get the pressure drop in single phase ,(p_1), as

$$p_1 = \int_0^{\lambda} \left[\frac{1}{A_x} \cdot \frac{dG}{dt} + \frac{f \cdot G^2}{2 \cdot D_h \cdot A_x \cdot \rho_c} + \rho_c \cdot g \right] \cdot dz \quad \text{-------} \ (86)$$

The perturbation in single phase pressure drop ,(P_1), can be found

by differentiating (86) with respect to G and λ and then adding both. After linearizing the resulting equation it reduces to

$$\mathbb{P}_1 = \int_0^{\lambda_0}\left[\frac{1}{A_x}\cdot\frac{d(\delta G)}{dt} + \frac{f\cdot G_0\cdot\delta G}{D_h\cdot A_x^2\cdot\rho_c}\right]\cdot dz + \left[\frac{f\cdot G_0^2}{2\cdot D_h\cdot A_x^2\cdot\rho_c} + \rho_c\cdot g\right]\cdot\lambda' \qquad \text{------ (87)}$$

Replacing δG by $G_0\cdot\,\mathbb{G}$ and then taking Laplace transforming in above equation,

$$\bar{\mathbb{P}}_1 = \left[\frac{\lambda_0}{A_x}\cdot s\cdot G_0 + \frac{f\cdot G_0\cdot\lambda_0}{D_h\cdot A_x^2\cdot\rho_c}\right]\cdot\bar{\mathbb{G}} + \left[\frac{f\cdot G_0^2}{2\cdot D_h\cdot A_x^2\cdot\rho_c} + \rho_c\cdot g\right]\cdot\bar{\lambda}' \qquad \text{-------- (88)}$$

Substituting $\bar{\lambda}'$ in terms of $\bar{\mathbb{G}}$ and $\bar{\mathbb{Q}}$ from (37),

$$\bar{\mathbb{P}}_1 = \Gamma_1(s)\cdot\bar{\mathbb{G}} + \Gamma_2(s)\cdot\bar{\mathbb{Q}} \qquad \text{------- (89)}$$

where

$$\Gamma_1(s) = \left[\frac{f\cdot G_0^2}{2\cdot D_h\cdot A_x^2\cdot\rho_c} + \rho_c\cdot g\right]\cdot\Lambda_1(s) + \frac{f\cdot G_0^2\cdot\lambda_0}{D_h\cdot A_x^2\cdot\rho_c} + \frac{\lambda_0\cdot G_0}{A_x}\cdot s \qquad \text{-- (90)}$$

and

$$\Gamma_2(s) = \left[\frac{f\cdot G_0^2}{2\cdot D_h\cdot A_x^2\cdot\rho_c} + \rho_c\cdot g\right]\cdot\Lambda_2(s) \qquad \text{---- (91)}$$

Hence resultant total single phase pressure drop perturbations transfer functions can be written by (89).

4.2.6 Analysis for two phase pressure drop perturbation :

The analysis for two phase pressure drop perturbation essentially follows approach as given in Lahey & Moody [7].

For the case of vertical, homogeneous two phase flow, the momentum equation is given by

$$-\frac{\partial p}{\partial z} = \rho_H\cdot\frac{\partial v}{\partial t} + \frac{f\cdot\rho_H\cdot v^2}{2\cdot D_h} + \rho_H\cdot g \qquad \text{-------- (92)}$$

Where ρ_H is the density of two phase mixture which is given by

$$\rho_H = \frac{1}{v_f + x \cdot v_{fg}} \quad \text{--------} \quad (93)$$

Substituting x from (64) into (93) and simplifying,

$$\rho_H = \rho_f \cdot e^{-\Omega \cdot \tau} \quad \text{------} \quad (94)$$

To get two phase pressure drop (p_2) integrate equation (92) with respect to z from ,z = λ, to ,z = L . Hence

$$p_2 = \int_{\lambda}^{L} \left[\rho_H \cdot \frac{\partial v}{\partial t} + \frac{f \cdot \rho_H \cdot v^2}{2 \cdot D_h} + \rho_H \cdot g \right] \cdot dz \quad \text{------} \quad (95)$$

Perturbation in the two phase pressure drop ($\mathbb{P}_2$) can be found by differentiating p_2 with respect to v and λ and then adding. Finally taking the Laplace transfer of resultant expression,

$$\bar{\mathbb{P}}_2 = \int_{\lambda_0}^{L} \left[\rho_H \cdot \overline{\left(\frac{Dv}{Dt} \right)'} + \frac{f \cdot \rho_{HO} \cdot v_0}{D_h} \cdot \bar{v}' + \left\{ \left(\frac{Dv}{Dt} \right)_0 + \frac{f \cdot v_0^2}{2 \cdot D_h} + g \right\} \cdot \bar{\rho}'_H \right] \cdot dz$$
$$- \left[\frac{f \cdot G_0^2}{2 \cdot D_h \cdot A_x^2 \cdot \rho_c} + \rho_c \cdot g \right] \cdot \bar{\lambda}' \quad \text{---------} \quad (96)$$

The next step in the analysis is to specify the various terms in the integration of (95) so that spatial integration can be performed.

First consider the two phase velocity perturbation, $\bar{(v')}$, -

Equations (46) and (49) can be combine to give

$$v(z,t) = v_1(t) + \Omega \cdot [z - \lambda(t)] \quad \text{--------} \quad (97)$$

After perturbating at a fixed z and Laplace transforming,

$$\bar{v}' = - \Omega \cdot \bar{\lambda}' + \bar{v}'_1$$
$$= - \Omega \cdot \bar{\lambda}' + \frac{G_0}{\rho_c \cdot A_x} \cdot \bar{\mathbb{G}} \quad \text{-------} \quad (98)$$

Next term to be considered is the two phase acceleration perturbation ,$\left(\frac{Dv}{Dt} \right)'$, -

combining equations (50) and (97),

$$v(y,t) = v_1(t) + \Omega \cdot y \quad \text{--------} \quad (99)$$

Now taking material derivative of both sides of equation (99) and combining the result with equation (53),

$$\frac{Dv}{Dt} = \Omega\cdot[\Omega\cdot\{z - \lambda(t)\} + v_1(t - \nu)] + \frac{dv_1}{dt} \qquad \text{------ (100)}$$

Perturbation in $\left(\frac{Dv}{Dt}\right)$ at fixed z will be given by

$$\left(\frac{Dv}{Dt}\right)' = -\Omega^2\cdot\lambda' + \Omega\cdot v_1'(t - \nu) + \frac{dv_1'}{dt} \qquad \text{------- (101)}$$

Now taking Laplace transform

$$\left(\overline{\frac{Dv}{Dt}}\right)' = -\Omega^2\cdot\bar{\lambda}' + \Omega\cdot e^{-s\cdot\nu}\cdot\bar{v}_1' + s\cdot\bar{v}_1' \qquad \text{------ (102)}$$

using (61) in (102)

$$\left(\overline{\frac{Dv}{Dt}}\right)' = -\Omega^2\cdot\bar{\lambda}' + \left(\Omega\cdot e^{-s\cdot\nu} + s\right)\cdot\frac{G_0}{\rho_c\cdot A_x}\cdot\bar{\mathbb{G}} \qquad \text{---- (103)}$$

The next term that must be considered is the two phase mixture density perturbation , $(\bar{\rho}_H')$.

Perturbating equation (94) and Laplace transforming,

$$\bar{\rho}_H' = -\Omega\cdot\rho_f\cdot e^{-\Omega\cdot\tau}\cdot\bar{\tau}' \qquad \text{-------- (104)}$$

using (62) in (104)

$$\bar{\rho}_H' = -\frac{\Omega\cdot\rho_f\cdot e^{-2\cdot\Omega\cdot\tau}}{v_{10}}\cdot\left[\bar{\lambda}' + \frac{e^{-s\cdot\nu}}{(s-\nu)}\cdot\left(1 - \exp\{(\Omega-s)\cdot\tau\}\right)\cdot v_{10}\cdot\bar{\mathbb{G}}\right] \qquad \text{----- (105)}$$

Integrating (58) and combining it with (99),

$$v_0(z) = \Omega\cdot(z - \lambda_0) + v_{10} + = v_{10}\cdot e^{\Omega\cdot\tau} \qquad \text{------- (106)}$$

So

$$v_0(L) = v_{10}\cdot\exp(\Omega\cdot\tau_{ex}) \qquad \text{------- (107)}$$

Combining (94), (96) and (106) yield

$$\rho_{H0} = \frac{v_{10}\cdot\rho_f}{v_{10} + \Omega\cdot(z - \lambda_0)} \qquad \text{------- (108)}$$

Now using (108), (107), (105), (103) and (98) equation (96) can be integrated then one obtains

$$\bar{P}_2 = \Pi_1(s) \cdot \bar{G} + \Pi_2(s) \cdot \bar{Q} \qquad \text{-------} \ (109)$$

where

$$\Pi_2(s) = \Lambda_2(s) \cdot \left[\left(\frac{f \cdot G_0^2}{2 \cdot D_h \cdot A_x^2 \cdot \rho_c} \right) \cdot (1 - \Omega \cdot \tau_{ex}) - \frac{f \cdot G_0}{A_x \cdot D_h} \cdot \Omega \cdot (L - \lambda_0) \right]$$

and $\qquad$ ------ (110)

$$\Pi_1(s) = \Lambda_1(s) \cdot \left[\left(\frac{f \cdot G_0^2}{2 \cdot D_h \cdot A_x^2 \cdot \rho_c} \right) \cdot (1 - \Omega \cdot \tau_{ex}) - \frac{f \cdot G_0}{A_x \cdot D_h} \cdot \Omega \cdot (L - \lambda_0) \right]$$

$$+ \rho_c \cdot \tau_{ex} \cdot \Omega \cdot (1 - s \cdot \nu) + \rho_c \cdot \tau_{ex} \cdot s + \frac{f \cdot \rho_c}{D_h} \cdot \Omega \cdot (L - \lambda_0)$$

$$+ 2 \cdot \rho_c \cdot \Omega^2 \cdot \frac{(1 - s \cdot \nu)}{(s - \Omega)^2} + \frac{\Omega^2 \cdot A_x \cdot \rho_c^2 \cdot g \cdot \tau_{ex}}{G_0} \cdot \left(\frac{1}{\Omega} - g \cdot \Omega \cdot \rho_c \cdot \tau_{ex} \right) \cdot \left[\frac{1 - s \cdot \nu}{s - \Omega} \right]$$

$$\text{-------} \ (111)$$

The above equations are adequate to appraise the hydrodynamics stability margins of a boiling system. For appraising the likelihood of density-wave-oscillation in the boiling loop, it is considered that static pressure is continuous around the closed loop, thus

$$\bar{P}_1 + \bar{P}_2 = 0.0 \qquad \text{-------} \ (112)$$

hence

$$\Gamma_1 \cdot \bar{G} + \Gamma_2 \cdot \bar{Q} + \Pi_1 \cdot \bar{G} + \Pi_2 \cdot \bar{Q} = 0.0 \qquad \text{------} \ (113)$$

or $$\bar{G} = - \left[\frac{\Gamma_2 + \Pi_2}{\Gamma_1 + \Pi_1} \right] \cdot \bar{Q} \qquad \text{------} \ (114)$$

This gives the stability criteria for the density wave oscillation.

4.2.7 Neutronic Transfer function (NT) :

A point-reactor model with single delayed neutron group can be written as,

$$\bar{Q} = NT(s) \cdot \bar{\$} \quad \text{-------- (115)}$$

where NT(s) is given by

$$NT = \frac{b\cdot(s + 1)}{s\cdot(s + b + 1)} \quad \text{-------- (116)}$$

here

$$b = \frac{\beta}{\Lambda\cdot\lambda_p} \quad \text{------ (117)}$$

and β = Delayed neutron fraction

λ_p = Decay constant of precursor

Λ = Average life of prompt-neutron

$\bar{\$}$ = Laplace transformed perturbation in reactivity

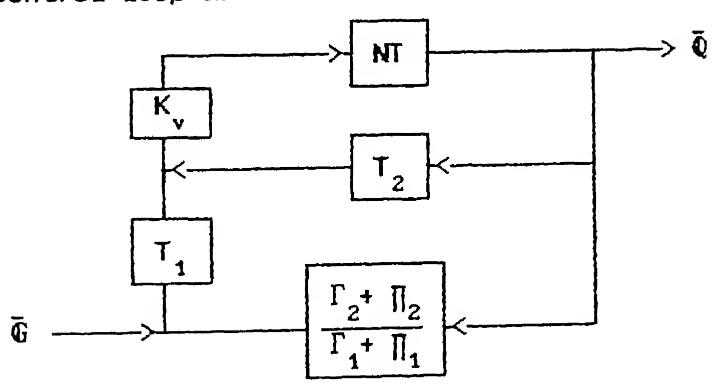
Also,

$$\bar{\$} = K_v \cdot \bar{\alpha}'_{avg} \quad \text{------- (118)}$$

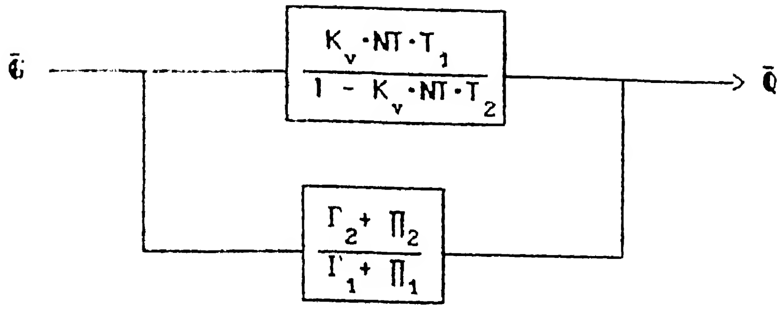
where K_v is the void reactivity feedback.

4.2.8 Analysis for decay ratio :

Now from equations (115), (118), (114) and (74) we construct a control loop as-



The above control loop can be simplify as



Hence gain transfer function G(s) is given by

$$G(s) = \frac{K_v \cdot NT(s) \cdot T_1(s)}{1 - K_v \cdot NT(s) \cdot T_2(s)} \qquad \text{-------} \ (119)$$

and feedback transfer function H(s) is given by

$$H(s) = \frac{\Gamma_2(s) + \Pi_2(s)}{\Gamma_1(s) + \Pi_1(s)} \qquad \text{---------} \ (120)$$

The overall transfer function will be

$$T(s) = \frac{G(s)}{1 - G(s) \cdot H(s)} \qquad \text{---------} \ (121)$$

Now applying root-locus technique for stability analysis of the system, the poles of the polynomial G(s)·H(s) was found. From these poles two complex conjugate poles which are closest to the origin are most sensible for stability analysis, were considered. Let these poles be $(m \pm i\,n)$

The behavior of the output with time corresponding to these poles will be of like as shown in figure(4.3).

where T is the time period of the oscillation

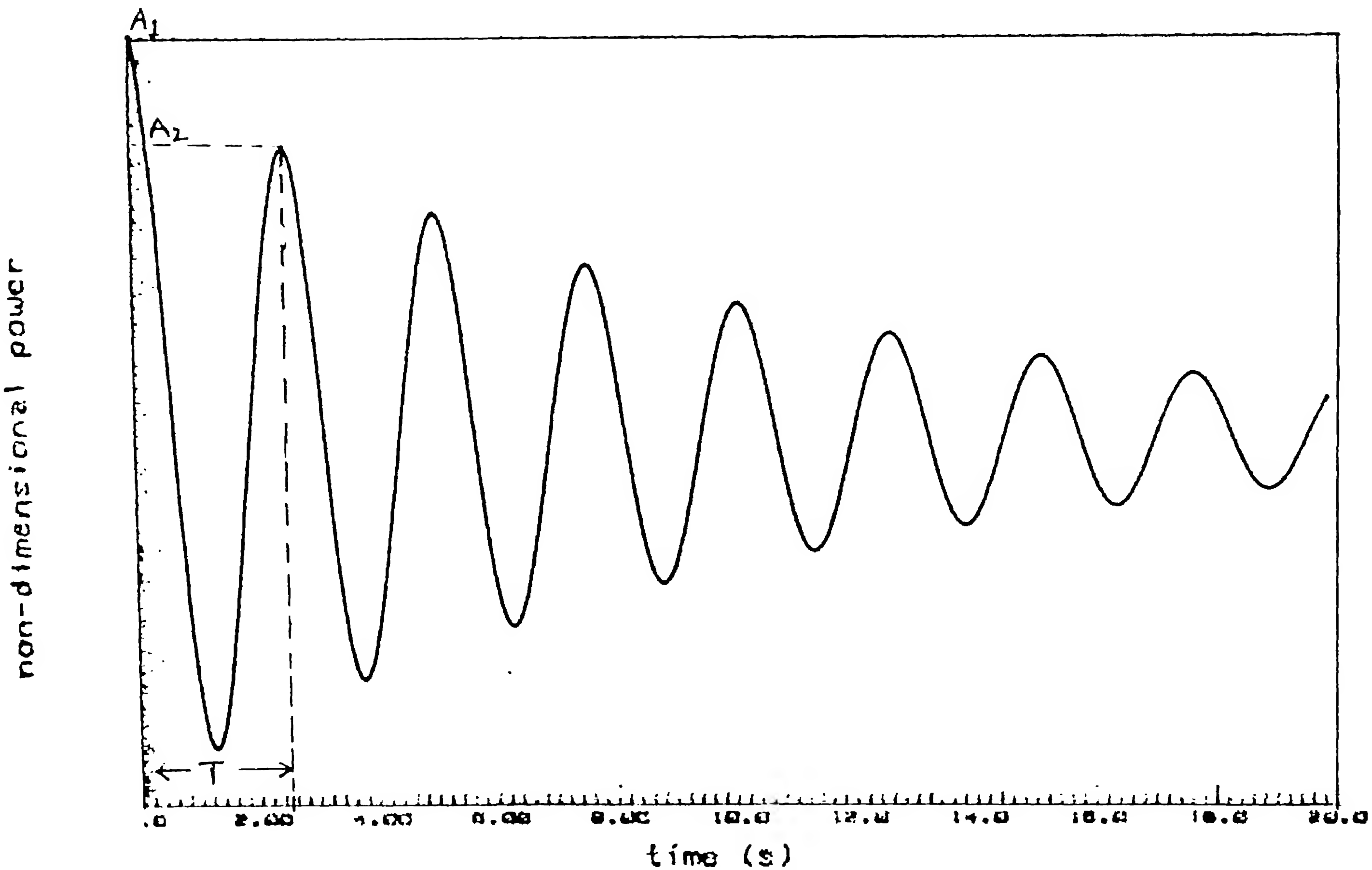
Now Decay Ratio is defined as

$$DR = \frac{A_2}{A_1}$$

$$= e^{-m \cdot T} \qquad \text{------} \ (122)$$

But $\quad T = \dfrac{2 \cdot \pi}{n}$

Hence



ig.(4.3) BEHAVIOR OF OUTPUT POWER with TIME

$$\boxed{DR = \exp\left(-\frac{2 \cdot \pi \cdot m}{n}\right) \qquad \text{--------} \ (123)}$$

4.3 Analysis for necessary riser height :

The analysis for necessary riser height essentially follows treaties given in El-Wakil [8].

The riser is an unheated extension of the core but usually with fewer walls, dividers etc. and of course, with no fuel elements so that there is less friction than in the core. The basic principle for the calculation of the riser height, in natural circulation BWR, is to balance the total pressure losses in the recirculation loop with the net driving pressure generated by the density difference between the water in the down comer and that in the channel.

The total pressure loss in the core is given by

$$\Delta P_{loss} = \sum \Delta P_f + \sum \Delta P_a + \sum \Delta P_{c,e} \qquad \text{------} \ (124)$$

where

ΔP_{loss} = Total pressure losses.

ΔP_f = Sum of the frictional pressure losses in the core, riser and down comer all computed in the direction of the flow

ΔP_a = Sum of the acceleration pressure losses , for this one considered only in the heated channel and ignore in the riser and downcomer since no large changes in density are encounter there.

$\Delta P_{c,e}$ = Sum of the pressure losses due to resistances to flow at abrupt area changes, for this one considered only at inlet and exit of the heated channel, and neglected the pressure losses due to the drag of submerged bodies such as spacers,

support plates , separators etc.

Now one wants to know the driving pressure. In case of no riser, the driving pressure is given by

$$\Delta P_d = (\text{ hydrostatic pressure in down comer}) - (\text{ hydrostatic pressure in channel})$$

$$= (\rho_{dc} - \bar{\rho})\cdot L\cdot g \quad \text{-------- (125)}$$

where

ρ_{dc} = density of coolant in the downcomer

and $\bar{\rho}$ is the average density in the channel and it is given by following expression

$$\bar{\rho} \cdot L = \rho_{nb}\cdot L_{nb} + \bar{\rho}_b\cdot L_b \quad \text{------- (126)}$$

here

ρ_{nb} = Density of coolant in the non-boiling section of the channel.

L_{nb} = Non-boiling height of the channel.

$\bar{\rho}_b$ = Average density of coolant in the the boiling section of the channel.

L_b = Boiling height of the channel.

ρ_{dc} and ρ_{nb} were assumed as equal to ,ρ_f, density of coolant in the liquid phase. The expression for $\bar{\rho}_b$ for the sinusoidal heat generation is taken as given.

Now to find the effect of riser on the driving pressure it is assumed that quality of the coolant in the riser is substantially the same as that at the channel exit. If the slip-ratio in the riser is assumed to be same as that of in the core, the void fraction in the riser will be constant and equal to that at the channel exit Hence the density along the riser will therefore be ... at the channel exit. It is then obvious

that the addition of a riser of height ,(L_R) , increases the driving pressure by the quantity $(\rho_{dc} - \rho_{ex})\cdot L_R \cdot g$. Where ρ_{ex} is the density of coolant at the exit of the channel.

Thus the driving pressure in the core of a channel with riser is given by

$$\Delta P_d = [\ \rho_{dc}\cdot(L + L_R) - (\bar{\rho} \cdot L + \rho_{ex}\cdot L_R)\ ]\cdot g \qquad ---- (127)$$

The exit coolant density ,(ρ_{ex}), is given by

$$\rho_{ex} = \left(v_f + x_{ex}\cdot v_{fg} \right)^{-1} \qquad ------ (128)$$

and can be calculated by knowing ,x_{ex}, the exit coolant quality. Now to get the riser height we equate the driving pressure with the pressure losses as

$$\Delta P_d = \Delta P_{loss} \qquad -------- (129)$$

By substituting ΔP_d from (127) and ΔP_{loss} from (124) one gets a equation with riser height ,L_R, as unknown parameter, so it can be solved to get the riser height.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Variation of Decay Ratio With Core Flow:

It was found that decay ratio is generally improved as the recirculation flow increases, this means that decay ratio decreases with the recirculation flow, and the margin of core stability increases. Figure(5.1) shows an example of the relation between the recirculation flow and the decay ratio for the two different core power density of 42 KW/l and 50 KW/l.The minimum recirculation flow necessary for the decay ratio less then 0.25 is also shown in the same figure. It was found that it increases with the increases in the core power density.

5.2 Variation of Minimum Recirculation Flow With Active Fuel Length:

The minimum recirculation flow is found by the condition of the decay ratio < 0.25 . It was found that it increases with the active fuel length. Figure(5.2) shows an example of the relation between minimum recirculation flow and the active fuel length for the core power densities of 42 KW/l and 50 KW/l.

5.3 Variation of Riser Height With Active Fuel Length:

The riser height necessary to produce the minimum recirculation flow was calculated according to the procedure set out in the chapter (4). It was found that riser height increases as the active fuel length increases. Figure(5.3) shows an example of the relation between the riser height and active fuel length for core power densities of 42 KW/l and 50 KW/l.

5.4 Variation of Riser Height With Core Power Density:

As both the minimum recirculation flow as well total power generation increases with core power density hence the necessary riser height to provide the minimum recirculation flow should increase and indeed it was found in this study. Figure(5.4) shows an example of the relation between the riser height and the core power density with keeping active fuel length as an independent variable.

5.5 Conclusion :

In this thesis the feasibility of the natural circulation BWR for higher power rates was studied. The thermal hydraulic requirements for increasing power rates were investigated analytically. It was concluded that natural circulation BWRs of powers > 1000 MW (electric) will be feasibile as far as core stability condition is concerns. The main conclusions are summarised below-

- As core flow increases , the core stability is improved.
- The minimum core flow to secure core stability can be mantained by increasing the riser height.
- The necessary riser height is approximately expressed as a simple function of power density and the active fuel length.

In fact according to the result of this study, the necessary riser height is mainly dependent on the active fuel length and the core power density.

The above results are much in agreement with the published results of Yasuso [1] and Kataoka [2] as it is clear from tables (5.1) and (5.2).

Table (5.1): Variation of decay ratio with coolant flow per bundle.

flow per bundle (Kg/s)	decay ratio			
	for q=42 KW/l		for q=50 KW/l	
	results obtained	results in ref.[1]	results obtained	results in ref.[1]
6.0	.729166	.61	.819128	.70
7.0	.527683	.45	.617643	.53
8.0	.376329	.32	.466294	.40
9.0	.271967	.25	.361939	.32
10.0	.198905	.18	.288884	.26
11.0	.146693	.12	.236676	.20
12.0	.108727	.09	.198714	.17
13.0	.080748	.07	.170738	.15
14.0	.059928	.05	.149920	.13
15.0	.044334	.04	.134328	.12
16.0	.032614	.03	.122609	.11
17.0	.023799	-	.113795	-
18.0	.017180	-	.107177	-
19.0	.012234	-	.102232	-
20.0	.008566	-	.098565	-

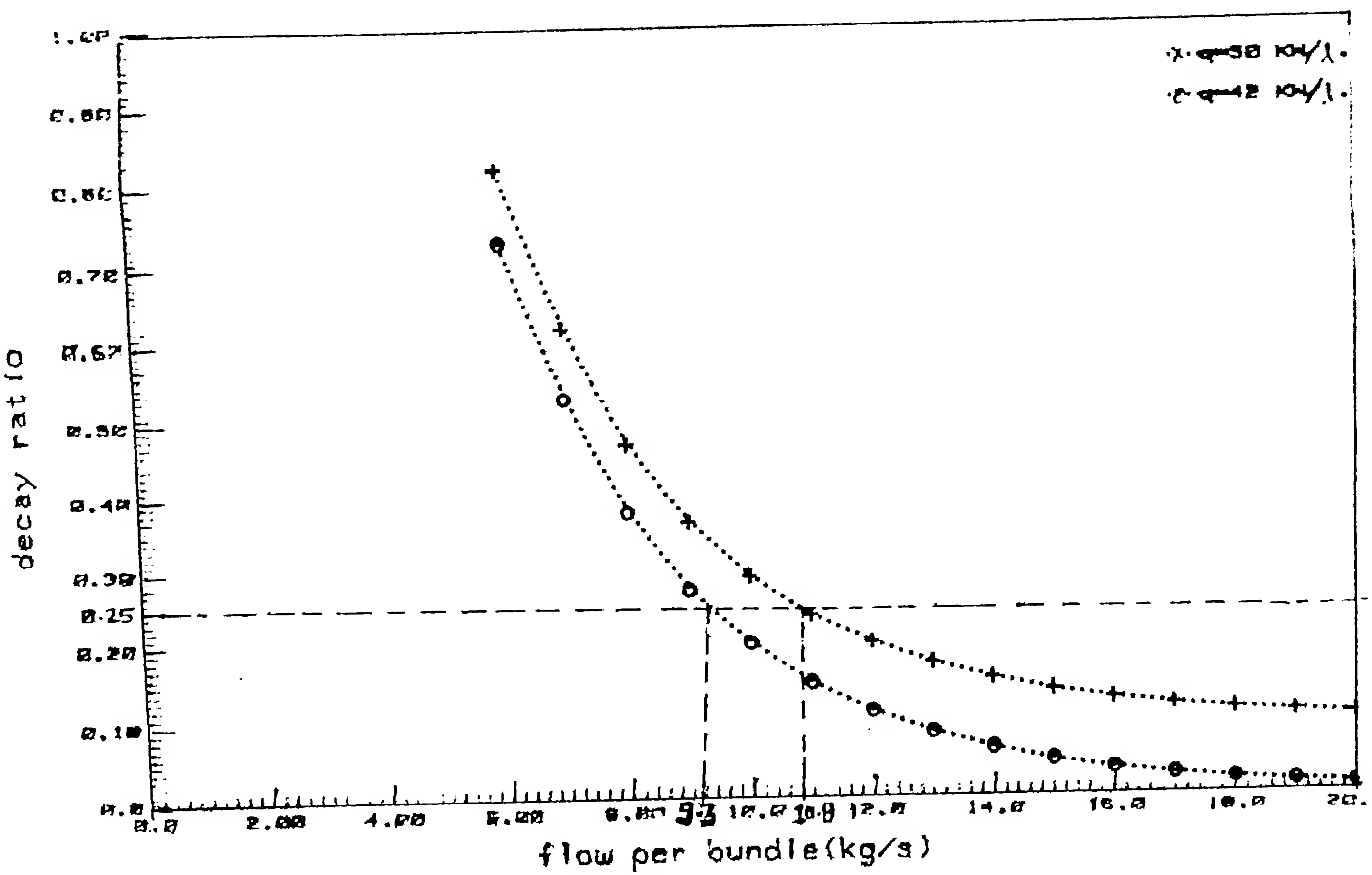


Fig.(5.1) DECAY-RATIO VS FLOW PER BUNDLE

Graphstar

Table (5.2): Variation of minimum core flow and necessary riser height with active fuel length.

active fuel length (in m)	for q = 42 KW/l			for q = 50 KW/l		
	minimum G per bundle	riser height (m)		minimum G per bundle	riser height (m)	
		results obtained	results in ref[1]		results obtained	results in ref[1]
2.3	3.75	2.4649	-	5.25	2.7219	-
2.4	5.32	2.8248	3.0	6.82	3.2549	3.5
2.5	6.73	3.2504	3.7	8.23	3.9147	4.5
2.6	8.04	3.7494	4.3	9.54	4.7184	5.8
2.7	9.30	4.3297	5.2	10.80	5.6842	7.0
2.8	10.56	4.9995	6.1	12.06	6.8309	8.0
2.9	11.87	5.7675	7.0	13.37	8.1784	9.1
3.0	13.28	6.6424	7.9	14.78	9.7476	10.2
3.1	14.85	7.6334	8.8	16.35	11.5603	11.3
3.2	16.62	8.7502	-	18.12	13.6397	-
3.3	18.66	10.0026	-	20.16	16.0099	-

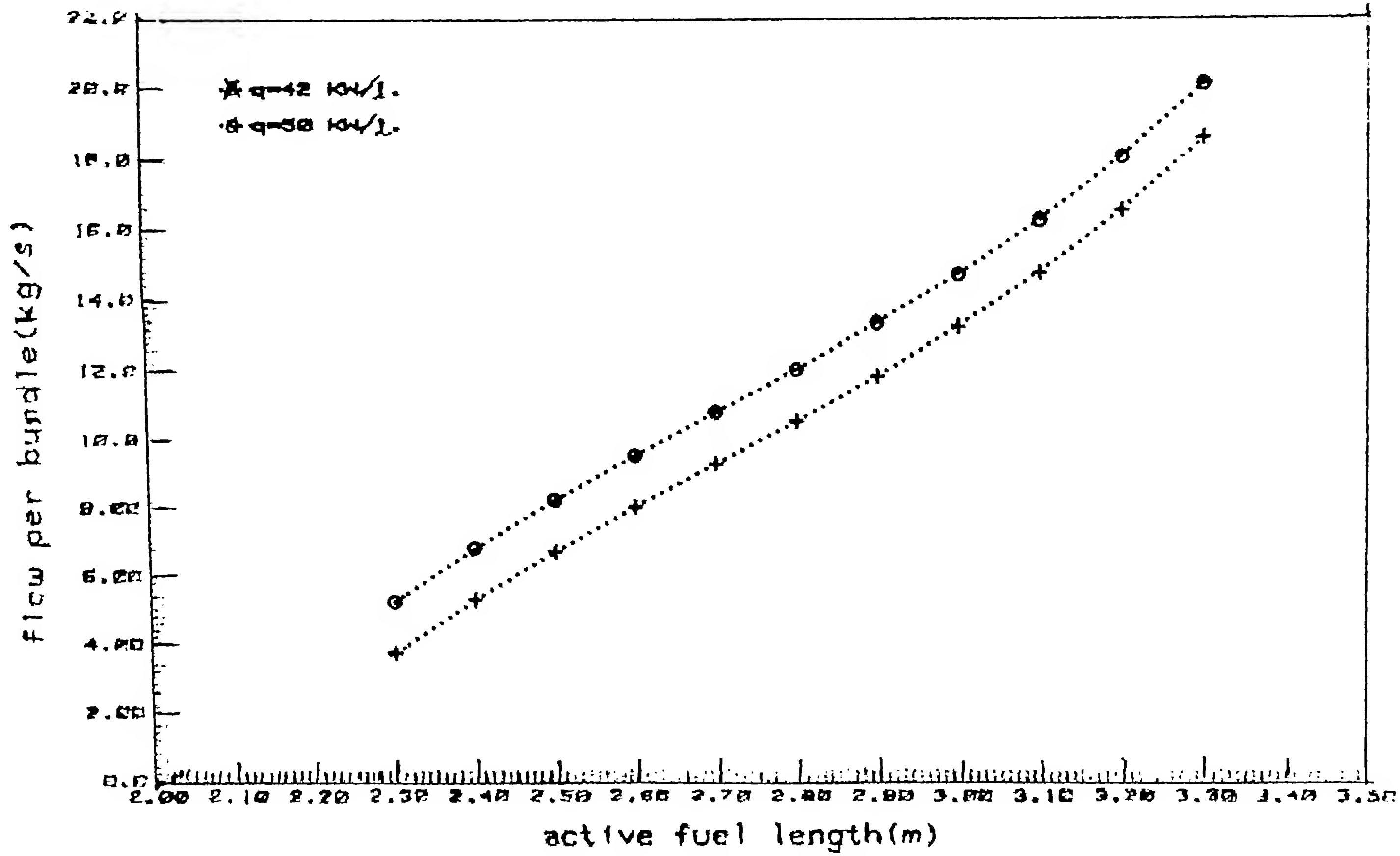


Fig.(5.2) COOLANT FLOW vs ACTIVE FUEL LENGTH

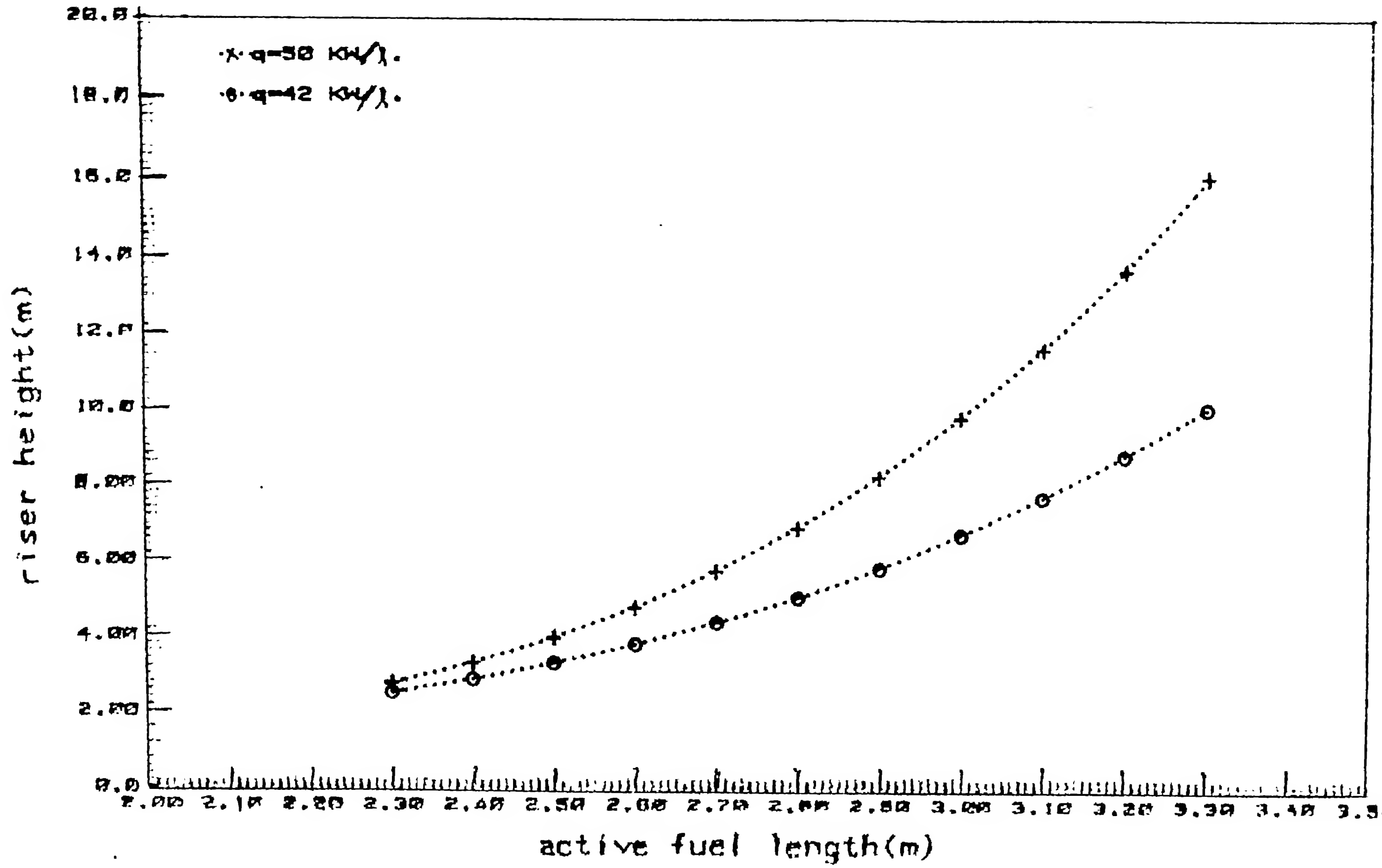


Fig.(5.3) RISER HEIGHT vs ACTIVE FUEL LENGTH

Graphstar

Table (5.3) Variation of riser height with core power density.

power density (KW/l)	riser height in (meter)			
	for L=2.4m.	for L=2.7m.	for L=3.0m.	for L=3.3m.
40.0	2.7596	4.2433	6.5382	9.8866
40.5	2.7655	4.2506	6.5475	9.8989
41.0	2.7856	4.2791	6.5848	9.9451
41.5	2.8053	4.3050	6.6152	9.9772
42.0	2.8248	4.3297	6.6424	10.0026
42.5	2.8443	4.3544	6.6696	10.0284
43.0	2.8490	4.3554	6.6624	10.0071
43.5	2.8692	4.3841	6.7005	10.0550
44.0	2.8899	4.4169	6.7492	10.1246
44.5	2.9115	4.4550	6.8122	10.2231
45.0	2.9341	4.5000	6.8931	10.3582
45.5	2.9580	4.5532	6.9958	10.5377
46.0	2.9834	4.6162	7.1247	10.7704
46.5	2.9949	4.6645	7.2441	11.0086
47.0	3.0239	4.7519	7.4386	11.3768
47.5	3.0551	4.8546	7.6746	11.8310
48.0	3.0889	4.9746	7.9585	12.3861
48.5	3.1254	5.1146	8.2981	13.0591
49.0	3.1650	5.2773	8.7023	13.8704
49.5	3.2080	5.4660	9.1814	14.8444
50.0	3.2549	5.6842	9.7476	16.0099

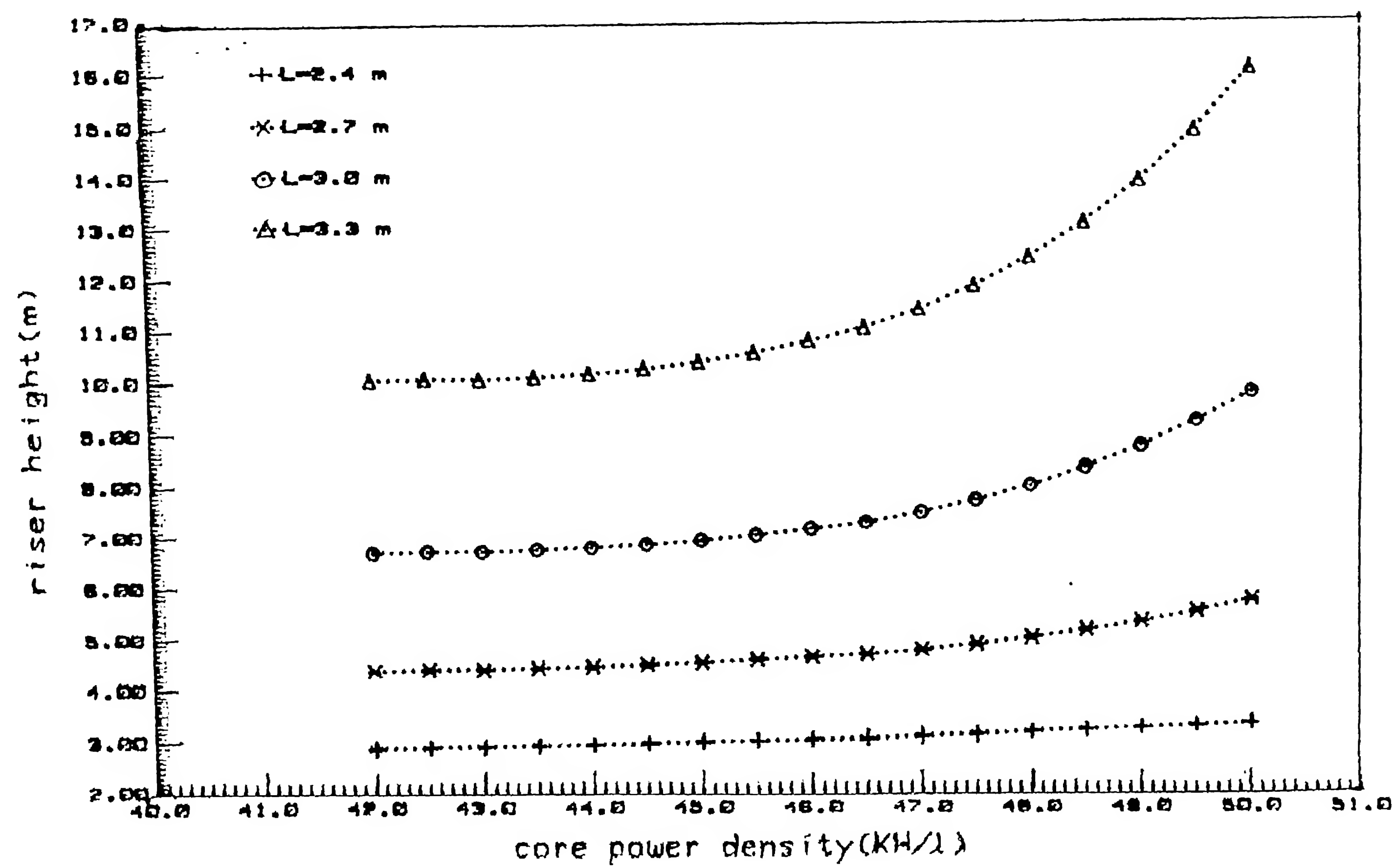


Fig.(5.4) RISER HEIGHT vs CORE POWER DENSITY

Graphstar

CHAPTER 6

RECOMMENDATION FOR FUTURE WORK

1. In the stability analysis, the perturbation in the inlet enthalpy of the coolant was neglected assuming that it will not make much effects on the results. But a more accurate results can be obtained by including inlet enthalapy perturbation in the analysis.

2. The uniform axial heat flux distribution was considered in stability analysis. However the peaking factor that gives conservative results, was used but it will be desirable to use the actual distribution that is the sinusoidal distribution for this analysis.

3. In calculating riser height one dimensional model is considered, but a more accurate results can be obtained using two-dimensional model with actual heat flux distribution.

4. In riser height calculation more accurate results can be obtained by considering pressure drop components of the bodies submerged in the core such as spacers, separators, plates, etc., which were not considered in present analysis.

5. The analysis has been done assuming uniform distribution of coolant flow among bundles and than considering the worst bundle but it will be better to consider the flow distribution corresponding to radial heat flux distribution so that stability criteria can be applied in each bundle.

6. The effects of parameters like reactor pressure, number of bundles, core diameter and many others on riser height which are not considered in present thesis , can be studied in future.

APPENDIX A

REACTOR SPECIFICATION

Parameter	Value
Operating pressure (p)	7.0 MPa.
Active fuel length (L)	2.3 to 3.3 m.
Fuel rod diameter (d)	12.3 mm.
Core diameter (D)	5.8 m.
Volumetric power density (q)	40 to 50 Kw/l.
Core inlet subcooling	33 KJ/Kg.
Fuel bundle type (M)	8 X 8 square.
Width of bundle (W)	0.13813 m.
Number of fuel bundle (N)	872
Local peaking factor (f_L)	1.2
Axial peaking factor (f_A)	1.5
Radial peaking factor (f_R)	1.4
Void-reactivity coeffiecient (K_v)	- 0.08 $ / % void.
Flow rate per bundle (G)	5 to 20 Kg/s.
Fuel material type	UO_2
Delayed neutron fraction (β)	0.0066
Decay constant of precursor (λ)	0.075 / sec.
Promt-neutron life time (Λ)	0.001 sec.

Table A.1 : Reactor Specification.

APPENDIX B

PROGRAM FOR CALCULATING DECAY RATIO

```
      REAL L,MU,K,KV,LEMDA,MT,MG,NBH,NBH1,K0,MF,NU,NN(10),ND(10)
      REAL NT1N(10),NT1D(10),NT2N(10),NT2D(10)
      DIMENSION Y1N(10),Y2N(10),Y1D(10),Y2D(10),T1N(10),T1D(10)
      DIMENSION T2N(10),T2D(10),P1N(10),P1D(10),P2N(10),P2D(10)
      DIMENSION F1N(10),F1D(10),F2N(10),F2D(10),GEN5(10),GEN6(10)
      DIMENSION GEN(10),GEN1(10),GEN2(10),GEN3(10),GEN4(10)
      DIMENSION HN(10),HD(10),GN(10),GD(10),GHN(10),GHD(10)
    { CONSTANT DECALERATION }
      HF=1258.18
      HG=2769.67
      DHI=33
      VF=0.0014
      VG=0.028
      P=1000
      RELOSS=0.1
      QV=42.E03
      D=12.3E-03
      L=2.7
      PF=10970
      CF=171.68E-03
      MU=2.48E-03
      F=0.025
      PC=750
      G=9.81
      BETA=0.0066
      GAMA=0.001
      LEMDA=0.075
      KV=0.08
      W=0.13813
      N=872
      M=64
      DC=5.8
      PIE=3.1414
      PR=0.35
      K=10.0
```

```
    DO 5 GPB=1,20
    MT=GPB*N
    HFG=HG-HF
    VFG=VG-VF

{  TO GET EXIT QUALITY OF STEAM }
    MG=TPG/(HG*RELOSS)
    XE=MG/MT

{  TO GET NON-BOILING HEIGTH }
    HI=HF-DHI
    NBH1=DHI/(DHI+XE*HFG)
    NBH=(L*ACOS(1.0-2*NBH1))/PIE

{ TO GET COOLANT FLOW VELOCITY IN NON-BOILING PART }
    AP=PIE*D
    AX=(W*W-M*PIE*D*D/4)/M
    AA=AP*L
    G0=MT/(M*N)
    V10=G0/AX
    QS=QV*D/4
    C0=(0.71+0.0001*P)-1
    K0=VF/(VG*C0*(XE+VF*(1-XE)/VG)**2)

{ TO GET HEAT TRANSFER COEFFICIENT }
    DH=4.0*(W*W-M*PIE*D*D/4.0)/(4.0*W+M*PIE*D)
    RE=PC*V10*D/MU
    H0=(K*0.23*RE**0.8*PR**0.3)/(1000*D)

    U=QS*AP*VFG/(HFG*AX)
    TEXIT=-(ALOG(V10/(U*(L-NBH)+V10)))/U

    MF=PIE*D*D*L*PF/4
    NU=PC*AX*DHI/(QS*AP)
    B0=BETA/(LEMDA*GAMA)

{ TO GET Λ(s) }
    Y1N(1)=-NBH*H0*AA
    Y1N(2)=-0.2*NBH*MF*CF
    Y1D(1)=H0*AA
```

```
   GEN2(3)=1
   CALL INT(GEN2,3)
   GEN3(1)=1
   GEN3(2)=-NU
   CALL INT(GEN3,2)
   GEN4(1)=E
   GEN4(2)=CC
   CALL INT(GEN4,2)
   CALL MULTY(GEN1,GEN3,GEN5)
   CALL MULTY(GEN4,GEN2,GEN6)
   CALL ADD(GEN5,GEN6,GEN1)
   CALL PROD(Y,Y1N,GEN4)
   CALL MULTY(GEN4,GEN2,GEN)
   CALL MULTY(GEN1,Y1D,GEN4)
   CALL ADD(GEN,GEN4,P1N)
   CALL MULTY(Y1D,GEN2,P1D)

{ TO GET NT(s) }
   NN(1)=B0
   NN(2)=B0
   CALL INT(NN,2)
   ND(1)=0
   ND(2)=B0+1
   ND(3)=1
   CALL INT(ND,3)

{ TO GET H(s) }
   CALL ADD(F2N,P2N,GEN1)
   GEN2=F2D
   CALL MULTY(F1N,P1D,GEN5)
   CALL MULTY(F1D,P1N,GEN6)
   CALL ADD(GEN5,GEN6,GEN3)
   CALL MULTY(F1D,P1D,GEN4)
   CALL MULTY(GEN1,GEN4,HN)
   CALL MULTY(GEN2,GEN3,HD)

{ TO GET G(s) }
```

```
      CALL MULTY(ND,T2D,NT2D)
      CALL PROD(KV,NT2N,GEN)
      CALL SUB(NT2D,GEN,GEN1)
      CALL MULTY(NT1N,NT2D,GEN2)
      CALL PROD(KV,GEN2,GN)
      CALL MULTY(GEN1,NT1D,GD)

{ TO GET G·H(s) }
      CALL MULTY(GN,HN,GHN)
      CALL MULTY(GD,HD,GHD)

{ TO GET DECAY RATIO }
      A=GD(7)/GN(6)
      B=(GD(6)-A*GN(5))/GN(6)
      POLE1=-B/A
      A=HD(7)/HN(5)
      B=(-HD(6)+A*HN(4))/HN(5)
      C=(HD(5)-A*HN(4)-B*HN(3))/HN(5)
      DD=B*B-4*A*C
      ALPHA=-B/(2*A)
      BETA1=SQRT(-DD)/(2*A)
      TP=(2*PIE)/BETA1
      DR=EXP(-ALPHA*TP)
      PRINT*,GPB,DR
5     CONTINUE
      STOP
      END

{ SUBROUTINE TO INITIALIZE A POLYNOMIAL }
      SUBROUTINE INT(POLY,N)
      DIMENSION POLY(10)
      DO 60 I=N+1,10
      POLY(I)=0.0
60    CONTINUE
      RETURN
      END

{ SUBROUTINE TO MULTIPLY TWO POLYNOMIALS }
      SUBROUTINE MULTY(POLY1,POLY2,PMULT)
```

```
      DO 80 I=1,10
      PMULTT(I)=0.0
      DO 70 J=1,I
      PMULTT(I)=PMULTT(I)+POLY1(J)*POLY2(I-J+1)
   70 CONTINUE
      PMULT(I)=PMULTT(I)
   80 CONTINUE
      RETURN
      END

{ SUBROUTINE FOR PRODUCT OF POLYNOMIAL WITH A SCALAR }
      SUBROUTINE PROD(X,POLY,PRO)
      DIMENSION POLY(10),PRO(10)
      DO 90 I=1,10
      PRO(I)=POLY(I)*X
   90 CONTINUE
      RETURN
      END

{ SUBROUTINE TO ADD TWO POLYNOMIALS }
      SUBROUTINE ADD(POLY1,POLY2,AD)
      DIMENSION POLY1(10),POLY2(10),AD(10)
      DO 95 I=1,10
      AD(I)=POLY1(I)+POLY2(I)
   95 CONTINUE
      RETURN
      END

{ SUBROUTINE TO SUBSTRUCT TWO POLYNOMIALS }
      SUBROUTINE SUB(POLY1,POLY2,SU)
      DIMENSION POLY1(10),POLY2(10),SU(10)
      DO 99 I=1,10
      SU(I)=POLY1(I)-POLY2(I)
   99 CONTINUE
      RETURN
      END
```

APPENDIX C

SUBROUTINE FOR CALCULATING RISER HEIGHT

```
{ SUBROUTINE FOR CALCULATING RISER HEIGHT FOR GIVEN FLOW PER }
{ BUNDLE, ACTIVE FUEL LENGTH, AND CORE POWER DENSITY }
  SUBROUTINE RISER(GPB,L,QV,RH)
  REAL L,M,N,MT,MG,NBH1,NBH,KR
  HF=1258.18
  HG=2769.67
  DHI=33.0
  VF=0.0014
  VG=0.028
  RELOSS=0.1
  D=12.3E-03
  F=0.025
  PC=750.0
  G=9.81
  W=0.13813
  N=872
  M=64
  PIE=3.1415
  S=1.9
  FR=0.025
  FD=0.025
  DC=5.8
  KR=0.5
  TPG=QV*(PIE*DC*DC*L/4.0)
  MT=GPB*N
  HFG=HG-HF
  VFG=VG-VF
  QT=TPG/MT
{ TO GET EXIT STEAM QUALITY }
  MG=TPG/(HG*(1-RELOSS))
  XE=MG/MT
{ TO GET NON-BOILING HEIGHT }
  HI=HF-DHI
```

```
NBH=(L*ACOS(1.0-2.0*NBH1))/PIE

{ TO GET FLOW VELOCITY IN NON-BOILING PART OF CHANNEL }
AX=(W*W-M*PIE*D*D/4.0)/M
G0=MT/(M*N)
V10=G0/(AX*PC)

{ TO GET HYDRAULIC DIAMETER OF COOLANT FLOW }
D0=DC*SQRT(1.0+KR)
DH=4.0*(W*W-M*PIE*D*D/4.0)/(4.0*W+M*PIE*D)

{ TO GET FRICTIONAL PRESSURE DROP IN CHANNEL }
SIE=S*(VF/VG)
AE=XE/(XE+(1-XE)*SIE)
R=(1+(1-AE)**(-1)+(1-AE)**(-2))/3.0
DPF=F*V10*V10*PC*(NBH+R*(L-NBH))/(2.0*DH)

{ TO GET ACCELERATION PRESSURE DROP IN CHANNEL }
X=(((1-XE)**2)/(1-AE)+XE*XE*VG/(AE*VF)-1.0)
DPA=PC*V10*V10*X

{ TO GET PRESSURE DROP AT ABRUPT AREA CHANGES }
A2=N*(W*W-M*PIE*D*D/4.0)
A1=PIE*DC*DC/4.0
DPC=0.7*PC*V10*V10*(1-(A2/A1)**2)
DPE=MT*MT*VF*(X+1)*((A1*A2)**(-1)-A1**(-2))

{ TO GET AVERAGE DENSITY IN THE BOILING PART OF CHANNEL }
C1=(QT-2.0*DHI)/(2.0*HFG)
C2=-QT/(2.0*HFG)
C3=SIE+(1-SIE)*C1
C4=(1-SIE)*C2
PE=AE/VG+(1-AE)/VF
PCE=PC-PE
PG=VG**(-1)
IF (C3**2.GT.C4**2) THEN
X=PIE*NBH/(2.0*L)
X=((C3-C4)*SIN(X)/COS(X))/SQRT(C3**2-C4**2)
X=1.0-2.0*ATAN(X)/PIE
X=(C1*C4-C2*C3)*L*X/(C4*(L-NBH)*SQRT(C3**2-C4**2))
```

```
ENDIF
IF (C4**2.GT.C3**2) THEN
X=PIE*NBH/(2.0*L)
X=(C4-C3)*SIN(X)/COS(X)
X=(X+SQRT(C4**2-C3**2))/(X-SQRT(C4**2-C3**2))
X=(C1*C4-C2*C3)*L*ALOG(X)/(C4*(L-NBH)*PIE*SQRT(C4**2-C3**2))
PCB=(PC-PG)*(C2/C4-X)
ENDIF

{ TO GET VELOCITY OF COOLANT IN RISER }
VE=4.0*MT/(PE*PIE*DC*DC)

{ TO GET VELOCITY OF COOLANT IN DOWNCOMER }
VD=4.0*MT/(PC*PIE*(DO*DO-DC*DC))

{ TO GET RISER HEIGHT }
A11=FR*VE*VE*PE/(2.0*DC)+FD*VD*VD*PC/(2.0*(DO-DC))
C11=FD*VD*VD*PC*L/(2.0*(DO-DC))
D1=PCB*(L-NBH)*G
B1=C11+D1+DPF+DPA+DPC+DPE
RH=B1/(PCE*G-A11)

RETURN
END
```

APPENDIX D

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